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**Social Formation of Mathematical Activity in a Japanese Mathematics
Classroom: “Revoicing” as a Unit of Analysis**

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SOCIAL INTERACTION AND LEARNING IN MATHEMATICS CLASSROOMS
IN AUSTRALIA, GERMANY, HONG KONG, JAPAN, SWEDEN, AND THE
UNITED STATES

(Organiser/ Chair: David Clarke, University of Melbourne, Australia; Discussant:
Lieven Verschaffel, Leuven University, Belgium)

Introduction

Recent research has a common and persuasive vision of mathematics classrooms as a socio-culturally mediated milieu. Different classroom cultures mediate different values with respect to mathematical activity. In everyday classroom practice, teacher and students coordinate the extent to which they participate in particular mathematical activities. The member's role in accomplishing mathematical tasks and innovation of certain discourse types varies across classroom practices.

The purpose of this research is to investigate features social formation of mathematics activity in a Japanese eighth-grade classroom. The investigation focuses on the ways that teacher and students coordinate their participating rights, allocate their obligation, and take responsibility for engaging in structuring structure of mathematical activity during classroom interaction.

The following discussion consists of two parts. First part involves an indication of the theoretical framework and the justification for the methodology used. Theoretical framework which is grounded on the Vygotskian-Bakhtinian perspectives is taken to the study of the social interaction in which the concepts of “activity” and “voice” plays a significant role. The second part involves description and interpretation of an episode that occurred in consecutive ten lessons on linear function which were audio-video recorded by Japanese LPS research leader (Prof. Y. Shimizu).

Theoretical Framework and Methodology

The investigation has been influenced by the writings of Russian psychologists associated with the Cultural-Historical Activity Theory (Leontiev, 1959; Vygotskii, 1982, 1983). Central to their formulations is the notion that human beings live in a socio-cultural environment transformed by the historically accumulated systems of artifacts (both conceptual and material) of prior generations. Epistemological assumptions inherent in socio-historical school involve “culture”, “guided

participation”, “mediation” and “appropriation”. By *social formation of mathematical activity* I mean a system consists of three components: *setting cultural (mathematical) motive; guided use of cultural mediational tools; and appropriation of mathematical concept as an object.*

If the social formation of mathematical activity in the classroom is to be analysed, *social* and *mathematical* dimensions as a whole have to be taken into account (Ohtani, 2002). Thus, we need a “unit of analysis”(Vygotskii, 1982).

In search of a methodology that will provide such a “unit of analysis”, I shall draw on “revoicing” (O’Connor & Michaels, 1996). By “revoicing” we mean *a particular kind of re-utterance (oral or written) of one’s contribution by another participant in a discussion.* The concept of “revoicing” seems to have theoretical underpinning of the concept of “utterance” of Mikhail Bakhtin. A concrete utterance of an individual articulating consciousness or personality is called “voice” (Bakhtin, 1981). Uttering is an activity that enacts different values, perspectives, intentions, and worldviews. In the formulation of an utterance a voice addresses or responds in some way to previous utterances and anticipates the responses of other voices. From its definition, “revoicing” itself entails social dimension.

For mathematical aspect of “revoicing”, I focus on “conditions and definitions” which I shall draw on the work of Imre Lakatos (1976). In his “*Proofs and Refutations*”, Lakatos portrays historical debates within mathematics about what a proof of a theorem represents, by constructing a conversation among a group of students that contains mixed within it many perspectives. In the midst of an argumentation, revised definitions and conditions are progressively introduced in light of refutations. In this sense, introducing and revising definitions and conditions are acts of “revoicing” which are indispensable and essential components of communal classroom mathematical activity. “Revoicing” definitions and conditions reflect participant’s mathematical motive, tool-use, and meaning which constitute communal classroom mathematical activity. In Bakhtin’s view, certain discourse type is used extensively and will tend to be privileged in a certain socioculturally situated setting. Therefore, innovation and use of a specific discourse type of “revoicing” in Japanese mathematics classroom allows us to investigate the specific structure of social formation of mathematical activity.

Research Findings

Social Formation of Mathematical Activity

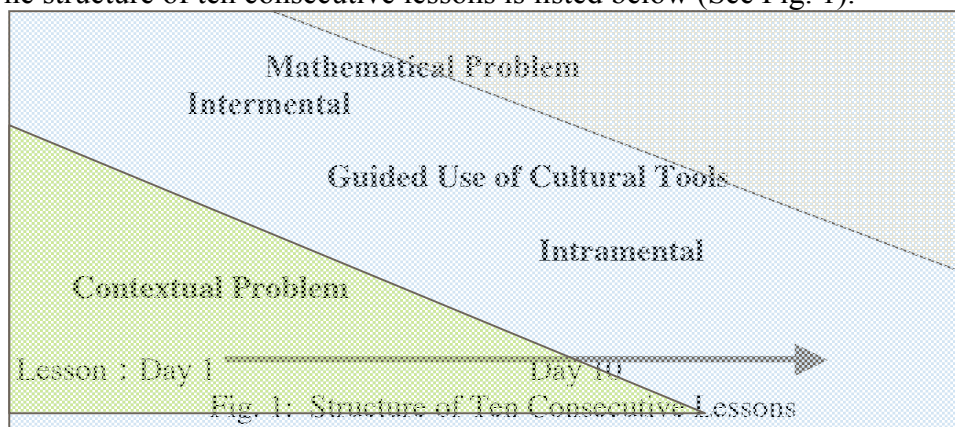
Ten consecutive lessons on linear function consist of three parts (Day1-3; Day 4-7; Day 8-10) each of which contains three components of mathematical activity: setting mathematical motive, guided use of cultural tools, and appropriation of mathematical concept. Topic, content, and cultural tools and their psychological functions are summarized below (Table 1).

Day	Topic	Content	Tool & Function
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1	Step Problem	Whole class activity to find dependent variables as dependent variable changes (discrete quantity: number of steps). Investigate relations between number of steps and length of its circumference. First, make a numerical table, then formula.	<ul style="list-style-type: none"> • From table to formula • Intermental: Reference, indicative, and communicative function
2	Ditto	Individual work to find other dependent variables. Make a formula from a numerical table.	<ul style="list-style-type: none"> • Transition from table to formula • Ditto
3	Ditto	Abstraction from concrete situation to formal aspect of algebraic formula.	<ul style="list-style-type: none"> • Formula • From intermental to intramental: First transition from referential to mathematical object
4	Paper Folding Problem	Individual activity to find dependent variables as independent variable changes (continuous quantity: length of paper). Investigate relations between independent variables and length of paper circumference (as dependent variables). Draw a graph.	<ul style="list-style-type: none"> • Table, formula, and graph • Intermental: Reference, indicative, and communicative function
5	Ditto	Presentation of individual work (numerical table and graph) to his/her classmates.	<ul style="list-style-type: none"> • Graph • Intermental
6	Ditto	Abstraction from concrete situation to formal aspect of numerical table, graph, and algebraic formula. Whole class discussion of properties each student found. Introduction of terminology of 'rate of change'.	<ul style="list-style-type: none"> • Table, Graph, and Formula • From Intermental to Intramental: Further Transition From Referential to Mathematical Object
7	Ditto	Same activity as day 6. Introduction of terminology of 'range'.	<ul style="list-style-type: none"> • Table, Graph, and Formula • Ditto
8	Relation between Formula and Graph	Represent formulae into graphs.	<ul style="list-style-type: none"> • From Formula to Graph • Use formulae to develop graphs as mathematical object.
9	Ditto	Whole class discussion of relations each student found between formula and graph. Introduction of terminology of 'gradient' and 'y-intercept'.	<ul style="list-style-type: none"> • Formula and Graph • Ditto
10	Practice	Work out problems on linear functions.	<ul style="list-style-type: none"> • Formula, Table, and Graph • Operate linear function as mathematical object.

Table 1: Topic, Content, and Cultural Tools and Psychological Functions

The structure of ten consecutive lessons is listed below (See Fig. 1).

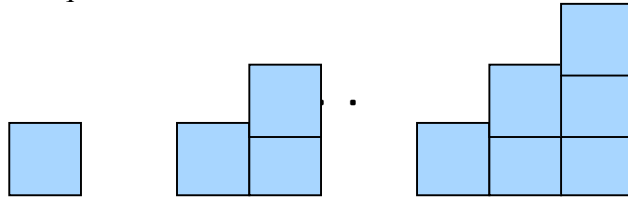


Setting Motive for Mathematical Activity

First part of the lessons (Day 1-3) consist in setting motive for mathematical activity. Teacher uses “contextual problem” to find dependent variables as dependent variable changes. By “contextual problem” we mean concrete situation which serve reference for through and model of mathematical relation. First contextual problem is “step problem”.

Contextual Problem 1 (Day 1)

Pile up some squares with sides 1cm, 1st, 2nd, 3rd as following figure.



- (1) Write down the figure of 4th and 5th.
- (2) What does it change if it's piling up 1st, 2nd, 3rd, 4th, and 5th steps

Through open-ended contextual problem the teacher initiate students motive for functional thinking. By *functional thinking*, we mean a method we use when there is a phenomenon which we want to control but cannot easily approach directly, we try to find out some other related phenomenon which are easier to treat. This method requires students to find and use *invariance* by systematically changing related variables (Shimada, 1990). Here is a protocol for setting a context for functional thinking:

T: Hello. Today, we're going to learn about new lesson. Just take it easy, ok? It is long time no see you guy! Did you change anything while this winter vacation?

S: Nothing.

T: Nothing?

TSUJI: I get good at skiing!

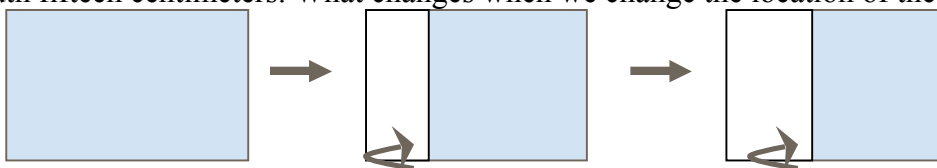
T: I gained weight this winter. Today's lesson is about “changing”. Did you change anything, guys? I think you did, huh? I gained weight, somehow. Today, I want you to think about “changing” through this figure.

Transition from Context to Mathematics

Second part of the lessons (Day 4-7) consist in progressive transition from context to mathematics. In the second contextual problem, student individually tries to find dependent variables as independent variable changes (continuous quantity: length of paper).

Contextual Problem 2 (Day 4)


Like the figure below, fold into a rectangle with length twelve centimeters and width fifteen centimeters. What changes when we change the location of the folds?



According to Leont'ev, a task is "a set of conditions". In this sense, the teacher progressively sets general and abstract task for the student. Task of Day 1 (Step Problems) concerns discrete variables. Task of Day 4 (Paper Folding) concerns continuous ones. The teacher expects students with different perspectives to find many kinds of dependent and independent variables. In the first contextual problem, the teacher asked students for finding many kinds of variables. In the second contextual problem, the teacher asked more advanced question. Here is a protocol for advanced functional thinking:

T: Yes, ok, well then I know there are more we haven't written down but let's think more later. Yes, well, uhh we thought about the changes that happen. Now what doesn't change? Some things don't change even when we fold this. **What doesn't change?**

From day 8, teacher proceed to general and abstract mathematical task.

Mathematical Problems (Day 8)	Mathematical Problems (Day 9)
Write graph of  .	Write graph of $y = 2x \square 7$.
Write graph of $y = x + 3$	Write graph of $y = \frac{1}{2}x + 3$
Write graph of $y = 2x$	Write graph of $y = \square 2x \square 5$

Guided Use of Cultural Tools

The principal role of symbolic devices, such as table, graph, expression is to overcome the extension in time, making the transitory constantly present and, at the same time, providing tangible means of communication about the phenomenon at hand. (Vigotskii, 1984; Nemirovsky & Monk, 2000). Among other tools, *functional algebraic expression* should be privileged and extensively used to investigate linear function. This expression represent external ratio which was not explicit in the numerical table and graph. During initial use, it has operational aspect for computing particular values. Later, the expression changes form one of the properties to defining character of linear function thus becomes a mathematical object.

For contextual problems, cultural tools function as a means not only to examine the problems but also to communicate their idea and conjecture in particular concrete situation to others. This function is called. "*intermental use*" (See Fig. 1). Here is a protocol data of teacher's support using cultural tools (Day 1).

T: Ok, so what do you think we have to do at first? For example, I record my weight everyday on my note. Then . . . What did you do if you want to do that when you are 7th grade? Remember? You write it down? We did the graph, right? You can tell me what you remember. You don't have to remember it if you can't. What else do you remember? Ms. Ogata, do you remember about it? Ms. Mawaridate?
MAWARIDATE: **A table.**

In the next protocol, teacher support student to use cultural tools in order for *inferring* unknown data (Day 3).

T: [Pointing out the table]Here, we don't have the fifth column. We don't have it here, but **can you assume what it would be?**

SAITO: Eight.

T: It would be eight. Alright, but how you figured it out? How about you, SAITO.

SAITO: The change had been, four, five, six, seven, and I thought the next one would be eight.

T: Humm... It would be eight. Yes, that's right. Let's think about this; how much would the result increases by one increase of the column. How is it in this case?

During the lesson of day 6, the teacher support students to think graph itself as a symbol of linear function.

T: Oh well, today, well, I copied some graph that you guys drew. It is small, but it's same. Number one... I will read it, ok? This table, on the top, and the memos and the graph, look at these things and write that something you found and something that you think it should be corrected. We wrote the table and we did the formulas last lesson. But we should look over it and correct it again, huh? And more characteristic. See? Write it down, please. Ok, let's take time. You can write whatever you want. Yes, whatever.

Such kind of activity allows students to recognize graph as a whole.

T: (Writes a graph on the chalkboard) One is like this type who marks with a many dots, and the other is like Uno who draws a line. What do you think, class? You are silent, aren't you? How about you, Yamagishi? Did you draw the line? (Yamagishi nod) Yes, look at the Yamagishi's one. [Shows the Yamagishi's hand-out with the camera] Well, you draw the line like this, Yamagishi. How come you draw?

YAMAGISHI: (Um, well, I draw the line because) there are small dots, much smaller one in the line.


Since the student recognize graph as an entity, they move to the problem of constructing corresponding formula of some linear functions.

T: To tell you the truth, we have the formula for that, so let me write it down. This formula is... Y equals to... what was that? Fifty-four minus two x. Like this formula, huh? To this formula, look at here. Please look at here, you guys. This formula, Miss, Sukanuma told us. What did you say? This one? Y is? Would you tell us that one more time?

SUGANUMA: Two x plus twenty-four.

Next activity consists in transition to solving mathematical problems. For mathematical problems, cultural tools function not only as a means to solve decontextualized problems but also an object representing linear function itself. This function is called. "*intramental use*" (See Fig. 1).

The purpose of mathematical task of day 8 and day 9 is to construct graphs from algebraic expressions. Such activity includes introduction of mathematical terminology for graph.

On day 8, students think about the meaning of a and b has in the expression of  and then write a graph. Teacher pose mathematical problems which contain some of defining characters of linear function. Students engage in solving these problems using cultural tools in order to find common properties among them. In the end of the activity, teacher introduce mathematical terminology of “gradient” and “y intercept”. When the student learn mathematical terminology, teacher “revoice” to student inarticulate utterance. Here is an example of such kind of “revoicing”.

T: Alright. Raise your hand if you can tell the differences in words, any differences that you can explain in words, only one. Okay, then please share it with the class. Yes, how about you, Niibori?

NIIBORI: Number, number five, when it's on the left side, it is at the very top, but if we think three as a border line, it comes to the very bottom, number two stays in the middle, and number one which is at the very bottom on the left side comes to the top when it crosses the border.

T: Goes to the top. Yes, goes to the very top. Um, if you had three lines like this, how can I say, don't you have any other words you use in your daily life, like slanted? For example, the stairs are rapid.

T: All right. And now each of us would like to express these things, like increasing by one, makes the other increase by two, without using the angle of degree. We call this gradation.

Conclusion: Teacher's “Revoicing” for Social Formation of Mathematical Activity

One of the specific characteristics of teacher (Ms. K)'s “revoicing” is its “publicity”. During classroom interaction, “revoicing” is extensively used by the teacher. Teacher not only reply to nominated individual student, but also address to all the students. This means that the teacher capitalize particular students contribution to address whole class in order to promote collective reflection. “Publicity of revoicing” is obvious during student independent work. In case, one of the students personally has difficulty with assigned task and asks question to the teacher while she is scanning independent work, the teacher do not reply to the sole student but loudly addresses to all the students. Thus, teacher's utterances are “half-private-half-public”.

Another important teacher's “revoicing” is its “intentionality”. Traditionally, Japanese teachers prepare *lesson plan* and their own teaching materials as routine practice. In the lesson plan, teacher expect student variety of responses for assigned task and has a plan to capitalize their contribution to formulate challenging problems and elaborate their solutions through collective argumentations. The teacher uses “revoicing” as a *social resource* to promote students' participation in communal mathematical activity. In the midst of the participation, revised definitions and conditions are progressively introduced by capitalizing students' contributions. Thus, teacher's “revoicing” has many functions in the formation of mathematical activity: It functions for setting mathematical motive, guided use of mediational tools, and

appropriation of mathematical concept by defining mathematical objects and introducing mathematical terminologies. In this paper, such kind of “revoicing” is named as “measured revoicing”.

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