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The Story of a "Teacher-Dominating" Lesson in Shanghai.

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Abstract

Teacher-dominating classrooms in Confucian-heritage cultures are always seen as an environment not conducive to learning in western countries. However, recent studies have shown that students learning in such classrooms can still give very good performances. Therefore, it seems that simple labels of "teacher-dominating" or "student-centred" have not explained the crux of the matter. Whenever a teacher tries to teach, she/he will try to bring the *object of learning* to the fore of the students' attention. What matters is how the object of learning is being taught to reveal its features or properties and how the students have experienced it. Our basic assumption is that a learner learns when she/he can discern the object of learning and variation presupposes discernment. Therefore, an analysis which depicts the dimensions of variation created in the lesson explains what students may possibly learn. In this paper, we will look at a single lesson in Shanghai from three perspectives. We first analyse how the object of learning is being taught and explained in the classroom from the perspectives of variation. Then, we look into the students' and teacher's interviews for their comments on what were seen as important, thus giving a triangulation of the story in the lesson.

Introduction

Teacher-dominating classrooms in Confucian-heritage cultures are always seen as an environment not conducive to learning in western countries (Biggs, 1996). Increasing number of comparative studies on Asian classrooms have produced interesting findings and a more comprehensive picture of the classrooms. Comparative studies of achievement carried out by the International Project for the Evaluation of Educational Achievement (IEA) have shown that students in Asian regions outperformed their counterparts in the western world (Mullis et al. 2000). The mismatch between unfavourable classroom image and outstanding achievement creates discussion on the so-called "paradox" which led to many studies about the teaching in classrooms and the psychological and pedagogical perspectives about Chinese learners and teaching (e.g., Leung, 2001; Paine, 1990; Stigler and Hiebert, 1999; Watkins & Biggs, 1996, 2001).

Following the IEA studies, Stigler and Hiebert (1999) carried out a video study which aimed to determine whether or not clearly distinguishable characteristics of teaching styles and methodology could be identified across cultures. An important result of their work is the claim of identifying teacher "scripts" which represent the practices of teachers in the USA, Japan and Germany This is a powerful image, but does a national teaching script really exist? Lopez-Real and Mok (2002) comments that the Japanese lessons described in the video study certainly do not 'fit' the Asian stereotype. In their experience of teachers in Hong Kong, and elsewhere in Asia, the popular pedagogy is closer to the German model reported in "Teaching Gap" where concepts are carefully explained but the 'transmission' mode is still dominant.

Various studies contribute an explanation from an understanding of the cultural roots. Some clarify that the Chinese concept of "practice" which does not necessarily

represent meaningless repetition fostering rote memorization. When learners revisit a concept, they are always possible to deepen their understanding of the subject matter within memorization (Marton, Dall'Alba and Tse, 1996 and Li, 2000). Leung (2001) discusses the East Asian mathematics in terms of six dichotomies. Two dichotomies are "product (content) versus process" and "rote learning versus meaningful learning". In the former, Leung explains that East Asian classrooms put emphasis on the mathematics content and the procedures or skills in dealing the content. In contrast, Western view stresses how the students arrive at the knowledge instead of the product. With respect to rote learning, this is always interpreted as "learning without thought" in Western views. However, memorization and repeated practice are accepted ways of learning in the East Asian culture.

Some studies look into the teacher and the teaching in the classroom *per se*. Paine (1990) describes teachers of Chinese in terms of the 'virtuoso model' in which the role of the teacher is mainly demonstrating and students are expected to follow. Ma (1999) found in her study that the Chinese elementary mathematics teachers showed a better understanding of the subject matter than the U.S. studies. In addition to reports on the teachers' capacity, the reports of the pattern of classroom interaction give a pluralistic picture. Mok and Morris (2001) present a classroom scenario with a combination of whole-class instruction and group work as a result of a sequence of curriculum reforms. Their description of the Hong Kong mathematics classroom is:

> Although the teacher-centred characteristics persisted, the teacher essentially played an orchestrating role of linking and developing a sequence of classroom activities... liveliness was maintained via the teacher's effort, skilful mode of delivery, questioning and receiving answers. (p.466)

Based on a theoretical framework of variation developed from the work of Marton and Booth (1997) and Runesson (1999), Mok (2000) analysed a demonstration lesson in a National Shanghai conference. Mok's work contributes to the understanding the Chinese mathematics teaching in two ways. First, the lesson is a teacher's attempt of a new pedagogical approach which is very different from the traditional transmission mode of teaching. This suggests that reforms in pedagogical approach are underway and the patterns of classroom are no longer uniform. Second, the work gives an example how the framework of variation can be used as a lens to understand how learning of a specific subject be made possible in additional to the set of generic environment measures such as whole-class interactive learning, group work, project work and so on.

Although teaching is likely to be comprehended in the context of cultural practices and the aforementioned has contributed to a better understanding of the nature of teaching in Asian regions, teaching is so complex that neither a simple label nor a script can be a fair description of the reality. Cultural characters may persist while the art of teaching in the classrooms is dynamic and evolving. Referring to the mismatch between the learning environment and achievement, it seems that simple social interaction labels of "teacher-dominating" or "student-centred" have not explained the heart of the matter. Research reports suggest that teacher-dominating is not necessary an equivalent to teacher-talk only or absence of interaction. A teacher-dominating lesson may be either good or bad and so may a student-centred lesson. To have a fair understanding of a lesson, we need data of the lessons and data from the relevant stakeholders of the lessons, i.e., the teacher and the students. We also need a

theoretical ground which can illuminate how a specific subject is handled in the lesson from a point of view of what is counted as significant to learning.

An objective of the paper is to contribute an answer to the question, "To what extent the label of 'teacher-dominating' can apply to a mathematics lesson in Shanghai?" The lesson to be discussed in this paper is taken from the Shanghai data set of the Learner's Perspective Study (LPS) the design of which provides an opportunity to triangulate a picture of a lesson from the teacher's, students' and the researcher's perspectives.

The Learner's Perspective Study

The Learner's Perspective Study (LPS), an international project led by Professor David Clarke at the University of Melbourne, is a video-study which involves a number of international partners, namely, Australia, Germany, Japan, USA, South Africa, Sweden, Philippines, Israel and China (Hong Kong and Shanghai). An important feature of the project is its documentation of the teaching of sequences of lessons, rather than single lessons like the TIMSS video study. Each class was recorded for a minimum of ten consecutive lessons after a familiarization period of recording. For each lesson, three cameras were used to capture the images of the teacher, the whole class and four focused students. A technique of on-site mixing of the images from two video cameras to provide a split-screen record of both teacher and student actions was used. The split-screen video was used for the video-stimulated recall student interviews carried out after each lesson in order to obtain participants' reconstructions of the lesson and the meanings that particular events held for them personally. Two students were interviewed after each lesson. Each teacher was interviewed three times during the whole period of recording. The technique of video-stimulated recall was used and the teacher could refer to the video of a lesson according to his/her own choice.

The lesson discussed in this paper took place in a school in Shanghai, one of the most fast-developing cities in China. The class was recorded for 15 lessons and the lesson here was the 7th lesson. It was a secondary-one lesson on "a system of simultaneous linear equations" which was a topic matching the Hong Kong curriculum of Grade-8. The teacher had more than 20 years of teaching experience, graduated in 1982 and awarded a "Lecturer in Secondary School" by the Shanghai Academic Title Appraisal Committee in 1992.

Theoretical Context

To situate my perspectives of learning and the theoretical context of this paper, I draw upon two books, *Learning and Awareness* by Marton and Booth (1997) and *Classroom Discourse and the Space of Learning* by Marton, Tsui, et al. (2003).

The point of departure is that a learner learns when she/he can discern the object of learning and variation presupposes discernment. The concepts of *the object of learning* and *variation* are discussed by Marton and Booth (1997) and further elaborated in the subsequent investigations of classroom teaching carried out by Marton, Tsui et al. (2003). As these are the key concepts in the theoretical context for this paper, I will explain the ideas briefly here.

According to Marton, Runesson and Tsui (2003), learning is always the learning of something which is called the object of learning. The authors refer the object of learning to a capability which has a general and a specific aspect. The general aspect

has to do with the nature of the capability such as remembering, interpreting and grasping. The specific aspect has to do with the subject on which these acts of learning are carried out, such as formulas and simultaneous equations. This object of learning may be often conscious for the teacher and may be elaborated in different degree of details. What teachers are striving for is the *intended* object of learning which is an object of the teacher's awareness. However, what is more important is how the teacher structures the lessons so that it is possible for the object of learning to come to the fore of the students' awareness. The researcher's description of the lesson from a specific research interest which tells how an object of learning appear in the lessons is called the *enacted* object of learning. Finally, what the student actually learns is the *lived* object of learning which is the object of learning from the student's view.

Having said the above, the standpoint for describing the enacted object of learning needs to base on an understanding of learning in order to make a fair appreciation of how students are made possible to learn in a particular situation. According to Marton et al. (2003) learning is a process in which we want learners to develop certain capability or a certain way of seeing or experience. In order to see something in a certain way the learner must discern certain feature of that object. It is important to differentiate between "discerning" and "being told". For the act of discernment, Marton and Booth (1997) give the following explanation:

> In order to experience something *as* something we must be able to discern its parts and relate them to each other and to the whole… To experience a particular situation in terms of general aspects, we have to experience the general aspects. These aspects correspond to *dimensions of variation*. That which we observe in a specific situation we tacitly experience as *values* in those dimensions. (p.108)

Experiencing variation is an essential experience for discernment, thus significant for learning. For example, one must have experienced a range of different colours (e.g., blue, red, orange) in order to see what "colour" means. The variation between the different colours thus forms a dimension of variation that helps learners to discern the meaning of "colour." Marton et al. (2003) further argue that it is important to pay attention to what varies and what is invariant in a learning situation. They give empirical examples of certain patterns of variation and some of these will be used in the analysis of the lesson in this paper. They are:

1. Contrast – a comparison between what the object is and what is not, e.g., "three" and not three such as "two" or "four".
2. Generalization – an experience of various appearance of the object, e.g., to grasp the concept of "three" in three apples, three monkeys, etc.
3. Separation – separating a certain aspect of the object from the other aspects. To experience this, one aspect must vary while other aspects remain invariant.
4. Fusion – an experience of taking several critical aspects into account.

Whenever a teacher tries to teach, she/he will try to bring the object of learning to the fore of the students' attention. What matters is how the object of learning is being taught to reveal its features or properties and how the students have experienced it. An analysis which depicts the dimensions of variation created in the lesson explains what students may possibly learn. Variation created in the lessons is not necessarily prescribed. A teacher may have created dimensions of variation without being aware of it. Therefore, the delineation of the dimensions of variation in the lesson represents

a researcher's perspective of the enacted object of learning, i.e., a potential space of learning.

A summary of the lesson

The lesson was about simultaneous linear equations in two unknown and last for 40 minutes. The teacher had prepared a powerpoint file containing all learning tasks, conclusion of the major parts and homework instruction, which he referred to from time to time during the lesson.

00:00:29

The teacher began by showing two powerpoint slides. The first slide showed the topic of the lesson "Method for solving a system of linear equations in two unknown (II)". The second slide showed a review of the substitution method learned in the last lessons. The review consisted of two lines which are translated below:

1. *Method for solving a system of linear equations: elimination by substitution.*
2. *The basic principle of elimination by substitution is to eliminate one unknown, to change from two-unknown to one-unknown in order to solve the equations.*

Then the teacher began by asking the class to solve a word problem mentally. Here is the translation of the problem shown on the screen.

Task 1

> *Xiu-min and his family went to Beijing for a holiday. They booked 3 adult tickets and 1 student tickets, costing a total of 560 dolllars. His classmate Xiu-wang learning this decided to join Xiu-ming's family for the trip. Consequently, they bought 3 adult tickets and 2 student tickets, costing a total of 640 dollars. Please calculate the cost of 1 adult ticket and the cost of 1 student ticket.*

The teacher invited the students to answer the question and explained the method. Two students (Dora and Eva) gave the answer with an arithmetic method. Then, the teacher asked the student to represent the problem in terms of two equations $3x+y=560$ and $3x+2y=640$. Referring to the students' earlier answer, the teacher explained that the arithmetic method was equivalent to subtracting one equation from the other and that this was the elimination method which was different from the substitution method learnt in the last lesson.

00:07:31

Then the teacher showed the next example. and asked the student to try allowing them to discuss with their neighbour.

Task 2

$3x+2y=8$ and 3x_2y=4

After about 3 minutes, the teacher asked two students to tell the class their methods. Doris subtracted the equations whereas Felix added the equations. The teacher then elaborated that the method was called "elimination by addition and subtraction" which was the topic of the day.

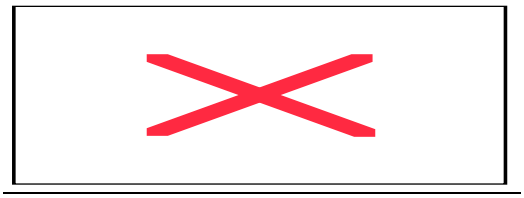
00:16:01

Then he asked the class to discuss why they could add or subtract the two equations and their criteria for whether they would add or subtract the equations. After about 1 minutes, the teacher asked the students to tell the results of their discussion. The students gave the answers, “the property of a system of equations”, “grouping like terms” and “the property of equations”. Receiving the three answers, the teacher asked the class which was the best reason. The class answered the property of equations which was immediately confirmed by the teacher.

00:19:29

Then the teacher showed Task 3 which had 4 pairs of equations and asked the class to determine whether they would apply the method of elimination by adding or subtracting the equations. The equations are listed here.

Task 3



The class answered the teacher’s question orally and the teacher asked the students to discuss and summarize based on these four exercises the criteria to apply addition or subtraction. The students then discussed in groups of four for about 1 minute. After this, the teacher asked the students to share their results. After two exchanges, the teacher drew a conclusion and then showed the conclusion on the screen which is translated here:

> [Summary 1: Conclusion of Task 3]
>
> If the coefficients of an unknown are opposite numbers, then we can add separately the two sides of the two equations directly. If the coefficients of an unknown are the same, then we can subtract separately the two sides of the two equations.

00:26:23

After this, the teacher showed an example and went through the procedure to solve for the solution with the help of powerpoint display.

Task 4

$5x-4y=12$ and $5x+y=7$

At the same time, the whole class read aloud the steps and the solution on the screen. Then, the teacher asked two groups of students to do one question and two other groups to do another in the textbook. He himself walked between the seats to see how the students worked.

00:31:23

After about 4 minutes, the teacher went through the work of two students by showing the work with the visualizer. In doing this, he asked the class to comment on the work and he himself also commented on the work. He also asked some questions about when to add or subtract the equations.

00:35:49

After this, the teacher showed the conclusion of the lesson and asked a couple of questions about the conclusion. The conclusion in the powerpoint is translated below:

[Summary 2: Conclusion of the lesson]

1. Solving a system of linear equations in two unknown by addition/subtraction elimination method;
2. If the coefficients of an unknown are opposite numbers, then we can add separately the two sides of the two equations directly. If the coefficients of an unknown are the same, then we can subtract separately the two sides of the two equations.
3. The basic principle of addition/subtraction elimination method is to eliminate one unknown, to transform from "two unknowns" to "one unknown" in order to solve the equation.

Then, the teacher gave the instruction for the homework and ended the lesson.

The Analysis

The arrangement of interaction

The lesson was a 40-minute lesson. Most of the time (76.4%), the lesson was carried out in a teacher-led whole class setting. These segments were either teacher-talk or students answering questions raised by the teacher. In between these whole-class segments, the teacher asked the students to work on a problem either individually (13.2%) or in small groups (10.3%).

There are two kinds of arrangement which the teacher often uses: a simple teacher-led whole class interaction and a planned short-interval seatwork individually or in groups followed by whole class interaction. In all whole class interaction, the teacher asked frequent questions and students answers the questions. There were hardly any instances of students raising their own questions. All seatwork began with a clear display of a task shown on the powerpoint and the teacher's clear instruction such as "you should discuss with each other in groups of two". These seatwork intervals were all very short (about 1 to 3 minutes) but quite frequent. During the seatwork, the teacher walked along the aisles to look at the students' work and sometimes spoke to the students. From the video, all students were engaged in the given tasks. After the seatwork, the teacher always resumed a whole-class discussion about the work. Moreover, the students were consistently attentive and followed the teacher's instruction. There were neither inattentive behaviour nor off-task instances. This observation was true for this lesson as well as the other lessons for this class.

Based on the video and the teacher's interviews, the events in both cases are very much under the control and within the expectation of the teacher. To conclude, the social interaction is under the control of the teacher.

The enacted object of learning

To help understanding the enacted objects of learning in the lesson and to facilitate the comparison between the analysis and the teacher's perspective, I will describe in terms of variation three consecutive segments of the lesson, Tasks 1, 2 and 3 – the same segments which the teacher commented in the interview. In these episodes, the object of learning is the method of elimination by adding / subtracting the equations.

The word problem and the interactions are all means to an end, aiming to bring the method into the students' awareness. Even though the teacher might or might not be aware of the variation he created, these dimensions of variation described subsequently explicate a potential space of learning.

Episode 1: Task 1

The problem was first solved by a student Dora mentally and her method was explicitly given in public. Dora's description is translated here:

00:02:15:23 Dora:Um, three adult tickets and one student ticket, um...totally five hundred and sixty dollars, and then adding one more student ticket will become six hundred and forty dollars, deducting five hundred and sixty dollars by six hundred and forty dollars will be the price for one student ticket.

This is apparently an arithmetic method invented by Dora and described in her own words. The teacher immediately endorsed Dora's answer by repeating the answers in his own words:

00:02:49:00
T:The difference between the two is the price for one student ticket, which means the price for a student ticket should be six hundred forty dollars minus five hundred and sixty dollars, which is eighty dollars.

Comparing Dora's answer and what the teacher said, Dora's is close to a thinking-loud representation of how she got the answer whereas the teacher's version highlights the act of subtraction in Dora's method. I call this a variation in the representation of Dora's method.

After getting the answers of the word problem, the teacher asked the class to do the problem again with the method of equations. This is a request for a variation in method, i.e, a different way to solve the problem. Immediately, the method instead of the answers (the cost of the student and adult tickets) becomes important. By this, the question how to solve the equations comes to the foreground in some students' mind. In contrast to Dora's method, the teacher told the class explicitly that the method of equation was an equivalent of Dora's method, followed by showing methods of equations on the board:

00:05:17:27
T:Okay, so let's see, Dora has just said that the price for each student ticket is six hundred and forty minus five hundred and sixty actually...six hundred and forty minus five hundred and sixty is equivalent to deducting the two equations we've just set.
...
00:07:06:01 T: Students please check it, right, in the process of solving the equations..., we've the value of one of the unknowns by deducting one equation from the other, so students let's see, it's not the same as the elimination-by-substitution method we learned earlier.

In Task 1, we can see how the teacher creates a contrast between Dora's arithmetic method and the equation method. In other words, he creates a dimension of variation for how to solve the problem. In each case, the method is represented in two ways. Dora's answer was first given in her own words followed by the teacher's version. The equation method was explained by the teacher verbally with the support of written boardwork. In addition to pointing out the new method by subtracting equations, i.e., the object of learning, he mentions this new method in parallel the old method ("elimination-by-substitution"). Putting the new and the old method side by

side, we see another variation by contrast. In this latter case, the dimension is that of the method of solving equations.

In this way, the teacher brought the class attention to the objective of his lesson (i.e., the method of elimination) which is a new method different from what they have learnt in an earlier lesson.

Episode 2: Task 2

Immediately following the end of Task 1, the teacher asked the class to solve Task 2 (3x+2y=8 and 3x_2y=4) with this new method.

```
00:07:32:07 T:    Students please follow this method to work on the
exercises, please use the method as provided by the questions to
solve the equations. [T showing slides]
00:07:49:13 T:    The equations are three x minus two y equals
eight...the second one is three x minus two y equals four...,
students please first think about what you should do, then try on
your own...students sharing the same desk can discuss with each
other.
```

The coefficient of Task 2 is intentionally set in such way that students can either subtract the equations to eliminate "x" or add the equations to eliminate "y". This design in the task deliberately helps learners separate the coefficient from the other features of the equations.

In addition to his instruction asking students to apply the method in Task 1, he also asked the students to discuss among themselves to think about what they should do. When such an instruction is given together with the presentation of variation of coefficients, it is of course likely to find some students to do either addition or subtraction. Moreover, while keeping the method of elimination in Task 1 in the background, the teacher shifts the context of the lesson from the word problem to a completely algebraic context. By asking the students to think about "what they should do", we can say that the emphasis is still very much on the method. This emphasis becomes explicit in the whole class interaction immediately after this.

```
00:11:51:03 T:    Okay, we will ask students to tell us. How did you
do it? Dorris. [S answering orally, T writing down on the board]
00:11:57:27 Dorris:     To solve, equation one minus equation two,
...
00:13:35:15 T:    So by deducting the two equations, we can change
them into linear equations in one unknown. For this question, do you
have other methods?  Okay, Felix.
00:13:55:27 Felix:      Adding them together.
00:13:57:01 T:    Can you describe it. [S answering orally, T writing
on the board]
00:14:00:05 Felix:      To solve, equation two plus equation one,
...
00:15:19:23 T:    The first one, we use deduction to achieve the aim
of eliminating one the unknowns, reducing it to a linear equation in
one unknown.
00:15:27:29 T:    The second, classmate Felix used the method of
adding to eliminate one of the unknowns, to obtain a linear equation
in one unknown, this method we call it the method of adding,
deducting and elimination. [T wrote on the board.]
00:15:52:19 T:    That is what we have to learn today the method of
adding, deducting and elimination to solve the system of linear
equations in two unknowns.
```

In this episode, the teacher asked two students to tell the class their methods. Dorris suggested subtraction of equations and Felix suggested addition of equations. To round up the answers, the teacher compared the two methods by pointing out that both methods eliminated the number of unknown. By this act, the new method is shown to have a dimension of variation between addition and subtraction. These are seen as one method with variation sharing the same principle rather than two different methods for solving the equations. Comparing with Task 1, the events in Task 2 not only provide an example of application but also lead the class to a more powerful way of seeing the method of elimination or the object of learning.

The discussion of Task 2 did not end here. The teacher asked a further question which put the experience of Task 2 to the background and brought the new focus onto a conceptual aspect of the method of elimination.

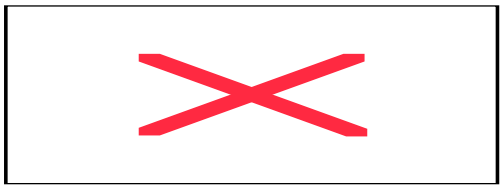
```
00:16:01:18 T:    So I have such a question, why you can use the
method of deduction for the two equations, why you can use the method
of addition for the two equations, what is the reasoning behind it?
How do you account for the adoption of such a step?
...
00:17:26:19 Fino: Characteristic of equations.
00:17:27:25 T:    Characteristic of equations. What kind of
characteristic of equations?
00:17:33:19 Fino: That is it can be added or subtracted among
equations.
...
00:17:55:03 Freda:      Combining items of the same kind.
00:17:56:10 T:    Combining items of the same kind. What do you mean
by that?
00:18:02:05 Freda:      That is adding or subtracting among items of
the same kind.
...
00:18:24:17 Daan: Feature number one for equality.
00:18:25:17 T:    Feature number one for equality. What do you mean?
00:18:28:09 Daan: For equality, adding or deducting a certain number
to both sides, the equality still holds.
```

The question invites justification for the validity of the procedure of adding or subtracting equation. In the students' answers (Fino's, Freda's and Daan's), we see that their attention has been successfully drawn to the properties of equations, the conceptual aspect of the method. The students' initial response in this part is very much like some secret codes to an outsider. Upon the teacher's request, these codes were explained by the students that they referred to properties of equation and equality which they probably had learned in other occassions. The open nature of the question solicits different students' answers which create a dimension. The creation of such a dimension brings the conceptual aspect of the method into the foreground.

Furthermore, the earlier classroom discourse on the procedural aspect and the latter on the conceptual aspect form a contrast which is a new dimension of variation of the elimination method that brings the understanding of the elimination method to another climax.

Episode 3: Task 3

Task 3 showed another way of how variation can be created. In Task 3, students are asked to determine whether to add or subtract the equation. The equations are given here:



The task is to determine whether to add or substract the equations without solving the equation. Therefore the focus is on the method but not on the answers for the equations. We have just seen how the dimension of variation of the coefficient of the equations lead to a tour towards the conceptual aspect of the method. Here we see the same type of variation of the coefficients again. The key to this task is to observe the coefficients of the unknowns in the equation. In this case, the same values or the opposite values appear in 4 different pairs of equations. No two pairs are the same yet they share some similarity. The students need to apply what they have just learned in Task 2 to determine whether they like to add or subtract. Such variation gives an opportunity for generalization. The criteria when to add or subtract the equations are brought to the foreground. In the later discussion of the criteria of for applying addition or subtraction of equations, the variation becomes explicit and is shared by the whole class. By this, the students observe what the teacher wants them to observe and the whole class arrive at the same conclusion.

00:23:07:11 T: And under what conditions can we use method of deduction to eliminate the unknowns? Eliminate the unknowns by the method of deduction?
00:23:21:23 T: Okay, students discuss in groups of four. [E discussing]
...
00:24:46:21 Donald: We've got (...), if the first and the second are contrary numbers then use method of addition, if not, having two numbers the same, then use method of deduction.
00:25:00:09 T: Have or not to have...sit down. Any supplementary information, Felix.
00:25:04:19 Felix: (...) For an unknown, the coefficient of one unknown, the coefficients of the unknowns, if they are the same, then use method of deduction, if they are contrary to each other, use method of addition.
00:25:20:07 T: Please sit down, for what Donald has said, look first at the coefficient, so if they are the same then use the method of deduction.
00:25:31:04 T: For contrary numbers use method of addition, so Felix has added that for an unknown, for an unknown. Let's think, if we have two unknowns, is it possible for them to be equal?
00:25:46:22 E: No. [Showing slides]
00:25:47:12 T: No, so we combine them together, that means, if the coefficients of an unknown are of contrary numbers we can use the method of addition to eliminate the unknown.
00:26:05:27 T: If the coefficients of the unknown are the same, we can use method of deduction to eliminate the unknown. We have to notice the criteria of the unknown.

A Summary

After the analysis of three consecutive segments of the lesson, I have to conclude that the teacher is very skilful in using varaition even though he may not be aware of it. When he wants to bring something to the foreground, he creates variation, for example, the alternative representations of Dora's answer and the alternative for how

to solveTask 1. The coefficients of the equations in Tasks 2 and 3 are so skilfully varied that students can experience separation or generalization to come to an awareness of the criteria for applying the appropriate method. Some of his questions pay an important role in creating a dimension of variation. For example, his question on why the method can apply in Task 2 solicits a number of students' answers concerning the property of equations leading to a dimension on the conceptual aspect. This way of bring out the conceptual aspect has kept the students' experience of the procedure in the background at the same time. This act has built a very natural link between the procedural part and the conceptual part of the addition/subtraction method. On the one hand, the quest for the conceptual aspect never appears to be appropriate without the earlier procedural part. On the other hand, the procedure appears to have no ground without the discussion on the conceptual aspect. In this particular example, the teacher makes the conceptual and procedural aspects be seen as a coherent whole by asking the students to reflect upon experiences at different intervals. This is very important in bringing about a conceptual understanding in mathematics. There is another similar example of such an act in the summary. The teacher asked the following questions:

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00:36:45:23 T:    Students please compare this with the method of
substitution we've learnt earlier and see if there's anything common
for the two methods?
00:37:03:27 T:    Remember, when using method of substitution to
solve the system of linear equations in two unknowns, what do you
think is common among the two, Emma.
00:37:23:19 Emma: Both of them start with getting one of the
unknowns, then, um, substituting this unknown into another equation
to find the other unknown.
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In this case the teacher asked the students to compare the method of the day with an old method retrospectively. In some way the question echoes what the teacher said at the end of Task 1 and creates a link between this lesson and the earlier lessons. More importantly the question puts the method of substitution and the method of addition/subtraction side by side for contrast so that both can be seen as parts of a coherent whole.

The teacher's perspective: Putting a theory into practice

We interviewed the teacher three times. He discussed two lessons in each of the first two interviews, one in detail and the other in brief. He claimed that these four lessons were all typical of his lessons. The lesson in this paper was the lesson in his second interview.

In the interview, the teacher was very aware of his personal style and he mentioned explicitly the application of a pedagogical theory by Professor Gu[1] which was also his school emphasis. With a reference to this theory, he was very happy with this lesson.

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[1] The Qingpu Mathematics Education Research Group, led by Professor Gu Lingyuan, was established in 1977 to carry out a ten-year study of reforms in mathematics teaching in Qingpu. The study included a 3-year survey, 1-year empirical selection of pedagogical principles, 3-year empirical study and 3-year implementation. Based on their empirical experience, they concluded 4 basic pedagogical principles: attitude, sequence, activity and feedback. With respect to classroom teaching, they proposed a 5-component framework:

1. Using problems as a starting point for teaching;

After these few lessons, you guys can see clearly that I have certain characteristics, or certain personal style. Basically I would set the situational questions, and then allow the students to try solving them, they would learn the knowledge in the process. Basically this is the format, so, I think it is very helpful in improving the ability of the students.

This lesson…actually I was practicing what our teacher Mr. Gu said in his book. I have been trying according to my personality, and I have improved myself. I have also arranged this lesson according to my own characteristics. So in this lesson, it had fulfilled every aspect that our school emphasized. For example, we had the situational questions, that is the first part as in the five parts mentioned by Mr. Gu, the second part is the activities for the students to try, I applied it as well. The third part is to summarize and conclude, I asked the students to do the summary as well. The fourth part is according to…oh, the third one is the activity for the students to try, I did it as well. The fourth part is to summarize and conclude. The fifth one is reflection. When teaching the students, I got some messages through reflection, in the students, in elimination by adding/subtracting. I would reflect at the appropriate time, and corrected the mistakes. This way the students can be more active, we can cultivate their interest, this is quite a good lesson.

In the interview, the teacher made 8 pauses in the video to explain in detail his rationale and reflections on 3 different parts of the lesson.

Part 1: A situational question, Task 1

The teacher called the first task in his lesson a "situational question" (wen ti qing jing), a special feature in Gu's theory.

I asked a question, that is, Siu Ming and his family is planning to take the train to Beijing to travel, in the question…there were two requirements, one of them is, the total amount of money for three adult tickets and one student ticket is five hundred and sixty dollars, the second one is that three adult tickets and two student tickets cost six hundred and forty dollars, I asked the students to find out the price of adult ticket and student ticket?

Actually I set a situational question as it can increase the interest of a student. He would find this kind of questions interesting, and he might want to try on his own. I also asked the students to calculate mentally before answering. So for this question, it was about elimination by adding/subtracting, so the students could answer very quickly.

The price of student ticket is six hundred and forty minus five hundred and sixty. Having such a result, which could help building a good foundation for

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2. Guiding students to develop exploratory activities;
3. Establishing variation in practice to raise the effectiveness of practice;
4. Summarizing to adopt into the knowledge structure; and
5. Modifying according to the fine categorization of teaching objectives.

Experimenting Group of Teaching Reform in maths in Qingpu County, Shanghai. (1991). Xuehui Jiaoxue (Learning to Teach). China: People Education Publishers. (In Chinese) pp. 3-13.

the students to experiment in future. So I find that, for this part, for me to introduce elimination by adding/subtracting, that is very important.

According to the teacher, the situational question has a number of functions: increase the students' interest, let the students be engaged quickly and to build a good foundation for future activity.

The teacher mentioned the importance of "foundation" several times in his interview. "Foundation" is definitely an idea in his focus. In order to make the question serve this purpose, he embedded two ways of solving the problem in the lesson. He knew that the question was simple enough for the students to calculate mentally (kou suan) and engaged them in the lesson quickly. As predicted the students gave the answers with the arithmetic method readily.

Lesson SH3.L07

00:02:15:23 Dora: Um, three adult tickets and one student ticket, um...totally five hundred and sixty dollars, and then adding one more student ticket will become six hundred and forty dollars, deducting five hundred and sixty dollars by six hundred and forty dollars will be the price for one student ticket.

Engaging the students in the situational question does not immediately guarantee a support for building a foundation. To serve this purpose, the teacher made a special attempt worked out by himself. He guided the class to solve the question once more by setting equations. In this way, he "translates the students' thinking into systems of equations", creating a guided discovery scenario.

Lesson SH3.L07

00:05:17:27 T: Okay, so let's see, Dora has just said that the price for each student ticket is six hundred and forty minus five hundred and sixty actually...six hundred and forty minus five hundred and sixty is equivalent to deducting the two equations we've just set.

It can also help build the foundation for future activities. So actually this question was an attempt, so that later on I would change this question into the format of equations. We would translate the students' thinking into systems of equations, so that they can easily discover that the two equations should be done by subtraction, in order to find out one of the unknowns. That is actually a new method to solve the system of linear equations, that is, elimination by adding/subtracting.

According to the teacher, the question which is his own design serves these functions much better be more welcome by his students when comparing with the questions in the textbook.

This question is different from the one we have in the textbook. The question in the textbook is about chickens and rabbits in a cage. …The students may find it boring. It seems…that the students are just following the teacher's thinking. So I give the students such a question for them to try, which is good for them to be more active in participating the lesson, that can make the students be more proactive in learning mathematics.

Part 2: The trial activity, Task 2

The next part the teacher explained in the interview is the second task in the lesson which is all algebraic. The teacher called this part the "trial activity" (chang shi huo dong) or an activity for the students to try, aiming to create an exploratory experience

for the students. This is an aspect of his school attempt in contrast to the old model of knowledge transmission.

> *I asked the students to use this method to solve this system of linear equation. The equations were three x plus two y equals to eight, and three x minus two y equals to four. Students could easily find that I used subtraction before, so they wanted to use subtraction to eliminate the unknown as well.*
>
> *Actually, according to my observation, most of the students used subtraction to elimination the unknown.*
>
> *A few of the students knew that using addition could also eliminate the unknown.*
>
> *This question demonstrated what I actually wanted to say about elimination by adding/subtracting and allowing the students to explore knowledge on their own. That is the second activity emphasized by the working experience of our school—activities for the students to try.*
>
> *That is we should throw out the old ideal that a teacher should only feed knowledge in class and nothing else. So…that is the second aspect of our school—activities for the students to try.*

Despite the mentioning of "trying" and "explore", the room for exploration is very limited and carefully controlled by the teacher. Consequently, the students' findings are within the teacher's expectation.

> *First, we can use subtraction to eliminate the unknown and it is not very troublesome, because by subtracting the right hand side, the result is four, it's not too large, and subtracting the left hand side is four y which is also not too large. I made the answers as simple as possible on purpose. The main aim is to teach them the methods, not on calculations. So this question should be arranged in this way, but why I do choose this way, one of the unknowns has equal coefficients, and the other unknown has contrary signs. The other purpose is to let the students think that, if the teacher can use the method of subtraction, then should they be thinking of using subtraction/adding as well? He can immediately find out that find out that he can use addition, oh, actually, some students did used addition to eliminate.*

Although the teacher took much lead in the content, he very much recognised the importance of student discussion. In another interview while commenting on a lesson on coordinates, he said,

> *Here I gave a question for the students to discuss whether this was on the x-axis or the y-axis, the coordinates and what were the characteristics of the coordinates of the point. During this discussion, students could get the conclusion through observation. That was good for the students to learn from each other, and it could let the students clarify the characteristics of the coordinates of the points on the two axes. So, I joined in their discussion from time to time. I would ask some questions for the students to think about.*

Part 3: Learning to summarise, Task 3.

According to the teacher, "to summarise" (gui na) is an important aspect in Gu's theory. He designed Task 3 specific for this purpose. Apparently the task looks like a

practice in which the students apply the new concept. However, the main objective is to provide a set of results for students to make observation and summaries.

> *In practice, this exercise only required the students to tell whether we should eliminate by adding or subtracting. And then I asked them two questions, under what conditions, the unknowns of the two equations of the system can be eliminated by adding? Under what conditions we can eliminate by subtracting? At that time I asked such questions, so that the students could discuss about it, and then made a conclusion. Due to the practice we had, we learnt how to make a conclusion. Actually students could improve their ability to summarize and to observe. So it is a very important part as mentioned by our teacher Mr. Gu. That is to integrate our knowledge into the education system, we have to know how to summarize, that's why I asked the students to summarize on their own, discuss on their own.*

This part of the lesson has two purposes. To let the students have a chance to summarize on their own can be an end itself and can also be a means to another end. The teacher obviously wanted the class to see the same summary which he saw as the proper conclusion.

> *After the students had their discussion, we exchanged ideas. At that time, if I discovered that the student's summary was not comprehensively done, I would give some hints and the conclusion could then be worked out by the joint effort of the teacher and the students. As they are only form one students, they haven't yet the ability to conclude well, they still need the help of the teacher.*

Lesson SH3.L07

The students' perspective: A focus on knowledge

We interviewed Franks and Franc individually after the lessons. We asked them to stop the video at pauses where they thought important and tell us what they were then thinking or doing. Franks made 3 pauses in the video-playback whereas Franc made 4 pauses. Their comments at the pauses are translated in table 1.

Table 1 The students' comments on the lesson

Time code	Franks	Franc	Remarks
00:05:14:00	I was looking at the blackboard…I was thinking about…what's the reason to set such a system of linear equations, …how to solve the system of linear equations.		Task 1 The teacher wrote the two equations on the board.
00:06:46:03		Because the teacher was telling us the ways to solve linear equations in two unknowns.	Task 1
00:12:29:00		Here the teacher was also teaching us the method of deduction, using the method of deduction to solve linear equations in two unknowns. .... Thinking why I couldn't do it,	Task 2

		(…) couldn't do it.	
00:19:18:05		Franc_ Because the teacher was telling us when we should use the method of addition to solve linear equations in two unknowns, and when we should use method of deduction. INT_ So when should we use the method of addition to solve? Franc_ Um, when the coefficients of one the unknowns are opposites, or they are contrary to each other, then we can add the two linear equations together directly. INT_ When should the method of deduction be used then? Franc_ If the coefficients of one of the unknowns are equal, then we can deduct the two linear equations with each other.	Task 3
00:26:16:00	I was looking at the screen, because this is…the concept of…elimination by addition/deduction, so I find it important.		The conclusion after Task 3
00:36:43:00		Because it is the summary of the lesson, it is also the most important part of any lesson.	The conclusion of the lesson
00:37:55:20	the teacher was talking about…the method of elimination by addition/deduction…with…with …similarities with the method of elimination that we learnt previously, so I think it is important.		The conclusion of the lesson

Both students said that they liked the lessons. Franks said that he liked the lesson because he learnt some concepts of mathematics which he though as important. Franc said that he liked the lesson because he gained knowledge. Therefore, learning some specific aspect of the subject are important to the students. Evidence of this inclination can also be found in their comments with reference to the lesson video.

Franks' comments show that he is critical and active in thinking. His first pause was where he himself started thinking how to solve the equation as soon as the teacher wrote the word problem (Task 1) into equations. The other two pauses were both when the teacher gave the conclusion of a certain part of a lesson. The second pause referred to the comparison between the method of addition and the method of subtraction. The third pause referred to the principle of the method. Both cases refer

to a conceptual aspect of the method, i.e. relationship between different methods or concepts.

Franc's comments showed that he is an attentive student. He put high value on knowledge and what the teacher said. Very often his comments began with phrases such as "the teacher was saying". These teacher-talks can be explaining method with examples elaboration of some concepts or conclusion of a part of lesson. He showed concern for moments when he noticed his own weakness, i.e., when he reflected upon why he could not do the question in Task 2. He paid attention to how to carry out and when to apply the method. Therefore, from Franc's perspective, nearly all parts of the lesson were important.

Summary

If one watches the video of the lesson without paying attention to what is said, based on the observation of the arrangement of interaction one will see all the events including the examples, the teacher's questions and the students' answers in fact follow closely the teacher's careful planning and expectation. One may conclude that the teacher is dominating in the sense that he controls what may happen and what the students may see or not. However from the teacher's perspective, the teacher certainly will not call his lesson "teacher-dominating". The teacher has a pedagogical theory purporting his planning and he sees his lessons very different from the traditional mode of knowledge transmission. Each part of his lesson is supported by a student-oriented rationale in his comments such as to help students develop a certain capability, to increase their interest, to motivate students to work on a certain task, to let students try and to provide a foundation for the students' further work. It is nearly possible to conclude that the teacher is very "student-centred" in his perspective. However, the teacher's conceptions of student-centredness are different from the western conceptions which show an emphasis on students' construction and collaboration and sometimes lead to a diversity of students' conceptions. In addition to the generic aspect, the teacher's objectives for his actions are intertwined with a specific aspect of the object of learning. Therefore, one can see how the teacher understands the object of learning (the method of elimination in this case), how he sees his way suitable for his students and helps the whole class see the object in his way.

Nevertheless, whether the label is "teacher-dominating" or "student-centred" really does not matter. The perspective of variation reveals a new facet of the lesson. With the question how the events in the lesson help students develop a certain way of understanding the object of learning, the basic means in the analysis is to look for dimensions of variation created in the lesson. However, the analysis is not an equivalent to a simple counting or identification of variation. In fact the variation described in the analysis is not exhaustive and never means to be. Only those which are seen to be helpful for the students to appreciate a specific or general aspect of the object of learning are included. Even more important, the analysis explains how certain arrangement of events or interaction between the teacher and students can possibly create dimensions of variation which give opportunities for the students see the object of learning in a certain way. The lesson is dynamic and the analysis aims to bring readers to go through the empirical events via an analytical lens. It represents a value judgement of how the events in the lesson can be seen as a possibility to bring about an understanding of the object of learning. The enacted object of learning in this lesson is very much controlled by the teacher whereas the students are only invited to

fill in the parts upon the teacher's request. With repect to the specific aspects of the method of elimination, the dimensions of variation have brought successively the procedural and conceptual aspects of the method into foreground and in one way or the other help students see the method as a part of a coherent whole.

Of course, we cannot expect that the teacher will use the term "variation" in the researcher's sense to describe his own lesson. His comments have shown that some dimensions are results of his advanced planning. For example, he let the students to solve the situation problem in two ways which he described as "translatng the students' thinking into systems of equations"; he designed the coefficients in Task 2 to let students discover that they can add the equations besides subtraction. He uses Task 3 to let students generalize after observing several examples. Therefore, it is also fair to say that the teacher indicates an implicit appreciation of the power of variation in his teaching.

Comparing the two students' comments, we see both similarity and difference. Their video pauses and their comments show how their interest in the lesson are different. Franks is more interested in conceptual aspects whereas Franc is interested in learning the application of the method. Despite their difference, they show a similarity that they are both keen on learning some knowledge. Both students pay great attention the mathematical content of the lesson analytically. They both capture some important parts of the lesson that many teachers will like their students to grasp. In one way or another, they capture both the procedural and conceptual aspect of the method. While they see the teacher-talk important, they also see moments when they themselves have an active thinking role important. Although the students in general appear to be obedient and follow the teacher's direction closely, they are in fact active in thinking. Active in thinking here refers to moments when they actively reflect upon the situation in the lesson which may either be a problem the teacher asks them to solve or a reflection upon what they themselves can or cannot do. This kind of active acts is implicit. It can hardly be visible if the students themselves do not point this out explicitly.

At beginning of the paper, I argue that simple labels such as "teacher-dominating" is an over-simplified label for Asian classrooms. To contribute to a fair understanding I attempt to present a triangulation of the researcher's, the teacher's and students' perspectives. By doing this, we see how the teacher with a strong pedagogical belief makes every effort to arrange his lesson so that his students will benefit most and understand the object of learning according to his own design. The enacted object of learning shows a delineation of relationship between procedure and conceptual aspect of the mathematics and that between the parts and the whole. Finally, we get evidence of students' keenness and active thinking moments in the students' interviews. All these are probably essential elements contributing to the success of a teacher-dominating lesson.

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