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## “ ‘cause Pepe and I have the same level of intelligence in mathematics’’: collaborative concept creation<sup>1</sup>

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Interactions between two Year 8 mathematics students were studied to identify factors that contributed to concept creation. Three cameras recorded the behaviours of: (a) the class; (b) the teacher; and (c) a pair of focus students. A mixed video image stimulated discussion in post-class student interviews. The use of non-routine tasks in a classroom environment that encouraged student autonomy led sometimes to student engagement in the exploration of discovered complexities and student creation of new concepts.

The concept of discovered complexity provided an analysis tool for my previous study of the nature and function of task complexity where a group of senior secondary calculus students were found to explore and resolve questions spontaneously formulated within the student group (*discovered complexities*) (Williams, 2000). The situation that triggered this student response was characterised by: (a) the provision of a task that provided opportunities for discovered complexities; (b) common student background within the mathematical domain opened up by the task; (c) a teaching approach that fostered student autonomy; and (d) sustained overlap in the student’s zones of proximal development (Vygotsky, 1978) during task completion. The discovery of complexities was accompanied by sustained engagement and exclamations of pleasure (Csikszentmihalyi & Csikszentmihalyi, 1992; Barnes, 2000; Williams, 2000), and the development of concepts initially unknown to all students involved in the interaction (*concept creation*). In describing the students’ activity, I have chosen to use the term *concept creation* as a subset of *concept construction* to emphasise students’ individuality in the problem finding and the selection of problem solving pathways. By this process, students develop concepts initially unfamiliar to all interacting students without mathematical input from sources external to the student group. As discovered complexities were associated with positive affective and cognitive factors, my present study explores the usefulness of discovered complexity as an analysis tool at another year level in the secondary school. The present study examines the interactions between two Year 8 boys who displayed evidence of engagement accompanied by conceptual change on several occasions during a sequence of area and perimeter lessons. The similarities and differences between the senior (Williams, 2000) and junior (present study) mathematics classrooms will inform my analysis of what characterises situations that trigger concept creation at the year eight level and do similar triggers operate at other levels in the secondary school? Answers to this question will inform task designers, teacher educators, and teachers studying classroom factors that could be varied to optimise the students’ learning environment.

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## Literature Review and Definitions

Vygotsky (1978) described the zone of proximal development as the difference between a student's present conceptual understanding and the understanding that could be attained in the presence of an expert other. As descriptions of situations that trigger conceptual change are pivotal to this study, I have formulated possible categories of conceptual change and described the type of assistance the expert has provided (Table 1).

Table 1

*Conceptual change categorised by the type of assistance from an expert other*

Category of conceptual change	Sub-category of conceptual change	Description of type of expert other
Concept reception	Concept intake: the concept is presented through an outside source and the learner makes no attempt to clarify it's meaning.	One-way interchange: The expert transmits their knowledge (e.g., traditional teaching).
	Concept acquisition: The learner questions to clarify meanings when an outside source presents a concept.	Two way interchange: The expert transmits their knowledge and the learner asks questions (e.g., peer tutoring).
Concept consolidation	Constructs previously presented but not fully understood by the students are explored.	Work on the selected task triggers <i>reciprocal scaffolding</i> (Holton & Thomas, 2001) where the questions the students (and the teacher) asked led to further concept elaboration for students.
Concept construction	Concept formation: learners gain continual feedback about the correctness of work as they explore and so build ideas.	Expert directed learner construction: e.g., suggesting, correcting and evaluating (Schaffer, 1996).
	Concept creation: the learners use mathematical ideas (initially unfamiliar to all learners involved) to spontaneously generate and explore focus questions (discover complexities).	Autonomous student activity triggered by the situation set up by the expert. During task performance the expert sometimes asks questions that facilitate student clarification, analysis, synthesis and evaluation (Barnes, 2000; Williams, 2000).

Conceptual change may occur when students work individually (Thornton, 1999), when a child interacts with an adult (Schaffer, 1996); when students collaborate to consolidate partially formed concepts (Holton & Thomas, 2001); and when students collaborate to create novel concepts (Barnes, 2000; Williams, 2000). Conceptual change may occur in the presence of an expert other or without the continual presence/participation of an expert other (Thornton, 1999; Barnes, 2000; Williams, 2000).

Krutetskii (1976) recognised, and my study of senior secondary calculus students illustrated (Williams, 2000), students with a high level of ability to solve unfamiliar challenging problems tend to explore many aspects of a problem (discovered complexities) rather than attaining fast closure of the task as set. The process by which conceptual change occurs as students work to solve unfamiliar challenging problems has been the focus of attention in recent years in both collaborative problem solving (Barnes, 2000; Williams, 2000; Clarke, 2001; Holton & Thomas, 2001) and solving of unfamiliar challenging problems by students working individually as illustrated in Thornton's (1999) study of primary school students using a limited supply of variously shaped blocks to build a bridge that fitted a set of constraints. Both Thornton's and Williams' students worked with dynamic tasks that provided the opportunity for students to focus their own exploration and autonomously decide what strategies to employ. The learning outcomes of both these groups were a function of the strategies they employed and the focus the students place upon their explorations. Both Thornton's and Williams' studies indicated these explorations precipitated concept creation.

*Non-routine Problems.* Non-routine problems (for the purpose of this study) are problems that rely upon the concepts developed in recently taught work but involve explorations for which students do not possess a 'taught algorithm'. These problems are intended to: (a) consolidate prior learning; (b) increase understanding of the newly developed concepts; and/or (c) extend the conceptual understanding of the students.

*Co-operative and Collaborative Pairs.* For the purpose of this study co-operative (as opposed to collaborative) pairs work together to assist each other but the pair do not share a common goal. For example, a student assisting another student who requests and directs help with a specific task. Collaborative pairs work together to achieve a shared goal. This 'working together' is characterised by student discourse leading to group creation of ideas.

*Discovered Complexity.* These complexities are not apparent at the commencement of the task but become evident during task performance (Williams, 2000). They possess two key features that meet M. Csikszentmihalyi's (Csikszentmihalyi & Csikszentmihalyi, 1992) stated conditions for flow; (a) student/s focus on a search to answer a question implicitly or explicitly formulated by the group or individual (intellectual challenge); and (b) this search encompasses mathematical ideas and concepts new to all group members.

## Methodology

This data is drawn from Australian data within the broader Learners' Perspective international study into the learning of Mathematics as viewed from the perspective of the learner. Each of the nine participating countries is presently undertaking studies of the classrooms of three teachers recognised by their school community to display 'good teaching practice'. The data from ten to fifteen successive lessons is collected by three video cameras that operate simultaneously in the classroom to display the actions of: (a) the class as a whole; (b) the teacher; and (c) a pair of focus students. Following the lesson, focus students take part in individual audio taped interviews that are stimulated by a mixed image of the video of the teacher (small insert) and video of the focus students. Photocopies of the students' work help focus the interview and inform the data analysis. The teacher participates in four similar video stimulated interviews (Clarke, 2001). Student

interview data and classroom discourse (Williams, 2000) were the primary sources used to locate and characterise situations that led to concept creation (Table 1). The interview question: *Did you learn anything new today?* was used to identify instances of conceptual change and the question: *Can you describe how that happened?* helped identify the type of conceptual change that occurred and the factors that may have triggered this change. The indicators used to identify high levels of engagement and to identify those factors that contributed to this engagement included student body language (Williams, 2000) and inferences made from student operation of the remote control in the video stimulated interview. As each student was asked to find and discuss the parts of the lesson that were important to that student, extremes in positive or negative affective factors were expected to form at least a part of the student focus in these interviews.

## Results and Discussion

Leon and Pepe's activity in two situations (a fortnight apart) were selected for analysis because Situation 1 provided evidence of collaborative concept creation (evidence from the video data and the field notes) and Situation 2 provided evidence of individual student participation in a situation where the teacher had organised students to work in pairs.

Table 2

*Characterisation of Situations that Triggered or Failed to Trigger Conceptual Change*

Students involved	Situation and Teacher Focus	Student Focus	Pedagogical Approach	Familiarity with Mathematics	Conceptual change?
Leon and Pepe	Situation 1. How many rectangles have a 38cm perimeter?	Relationship between the no. of rectangles ( to varying nos. of dec. pl.).	Open questions	Pepe: No Leon: No	Yes Concept creation
Pepe	Situation 2. Copy triangle and find the area	Construct a triangle when three sides are given; compass span too short.	Open questions	Pepe: No Leon: this was not of interest: used general case.	Yes Concept creation
Leon		The area of which triangle is easiest to find?	Teacher's task sequence	Leon: No Pepe unaware of this focus.	Yes Concept creation
Leon		Relationship between triangle and enclosing rectangle. Why?	Task sequence supported conceptual change.	Leon: No Pepe unaware of this focus.	Yes Concept creation.
Leon		Is a rectangle a special square?	Transmission	Leon: No Pepe: Yes	No

*During Situation 1 (see Table 2).* Mrs Milano's questions, and her point of task closure indicated she intended students to recognise: (a) a general formula  $L + W = 19$ ; (b) a reason why this formula works; and (c) that an infinite number of rectangles were possible. When Mrs Milano had presented the non-routine task in Situation 1, she assisted the class to develop ideas by writing student suggestions of specific examples on the board and altering these examples in response to class consensus about necessary changes. Once all the integer values for the dimensions of the rectangle had been supplied, Mrs Milano continued to ask for further responses stating several times: "the only restriction is that the perimeter has to be 38 cm". Mrs Milano's questions and the comments from other class members precipitated the rejection of student suggestions of negative numbers and zero (because they lacked meaning in relation to a rectangle). When all student responses appeared to be exhausted, Mrs Milano commented: "I am suggesting that there are other solutions". At this stage students began to provide decimal dimensions and some students began to recognise and articulate possible patterns. Mrs Milano asked why these pattern existed. The discussion continued until the class began to recognise an infinite number of solutions were possible. At this stage, Mrs Milano began to introduce the next task and several minutes later, Leon and Pepe's questions and comments to Mrs Milano commenced. As Pepe and Leon were not the focus students during Situation 1, the following evidence is drawn from my field notes, the video data, and student interviews (two weeks later). Both students appeared to be building ideas together in the few minutes before Leon asked Mrs Milano: "Do you know the answer if there was only one decimal place or ...?" Fragments of Leon and Pepe's interchanges with Mrs Milano have been used to infer the process of concept creation had commenced; students described their newly forming ideas in everyday language (Reeve & Reynolds, 2001): Leon "Nine hundred and ninety". Pepe "Nine thousand nine hundred and ninety" [simultaneously]; Leon "and so on and so on and so on"; and Pepe "it just keeps on going and going and going". Leon recalls some generalities of the pair's concept creation (in his interview two weeks later):

... and then if you go to two decimal places you can only have so many different decimal places for that number and like it kept building and building and building and you could do it [pause] um you sort of found a formula for it [pause] a way to [pause] categorise it into like different numbers

During the exchange with Pepe and Leon in Situation 1, Mrs Milano simultaneously continued to organise the whole class for the next task (dealing out string; requesting a student draw a circle on the board; responding to student questions, praising the student's circle; and recalling an historical anecdote about a well known Italian drawing a circle). Examples of Mrs Milano's responses to Pepe and Leon included: "Let me think about it. Can I think about it and you can as well?" "So you've got two decimal places. Why plus 90? [pause] Oh you've got the single decimal place plus the whole number?" "Well, you tell me. Would there? Why?"; "Yes there is an answer and I just can't think ...". "It depends too [pause] that's a really interesting question. Is 10 and 9 different from ten point zero and nine point zero and is that different from ten point zero zero and nine point zero zero?" Even though, at the time, Mrs Milano did not know the solution to the problem posed, her behaviour was consistent with her teaching philosophy (as explained in the teacher interview) and consistent with her usual behaviour to encourage students to develop ideas for themselves. The high degree of association between student engagement

and concept creation can be inferred from: (a) Leon and Pepe's high level of engagement prior to the interaction with Mrs Milano; (b) the joint and sustained contribution of both students to the discussion with Mrs Milano (even though the next class task had commenced); (c) the exclamations as each student recalled Situation 1 in their interviews two weeks later, and (d) Leon's description of this and other collaborative experiences:

Um Pepe and I are sort of [pause] at the same [pause] level of intelligence as well as like level of [pause] you know friendship and stuff [pause] that's why we work really really well together. 'Cause n- it's not just one of us doing all of the work [pause] and one of us just getting all of the answers and the other one sort of thinking 'oh what's going on here?' ... it's almost like we combine [pause] what we are thinking about and we sort of come out with the same answer.

Leon explained the *same level of intelligence* to mean to work at the same pace and to understand new ideas at the same rate. Leon thought he and Pepe had different levels of intelligence in other subjects. Table 2 (last three columns) shows: (a) the teaching approach; (b) student familiarity with the mathematics; and (c) the type of conceptual change.

*During Situation 2 (Table 2).* Mrs Milano drew three triangles of different shapes on the board (one right angled triangle) and allocated student pairs to find the area of particular triangles. Leon and Pepe failed to reach consensus about the level of generality with which they would approach the teacher's task so the students worked individually formulating their own idiosyncratic foci. Pepe was determined to use his own method (constructing the triangle and counting the squares) to find the area of the triangle. The construction of a triangle when three sides are known was unfamiliar to Pepe (student interview). Leon recognised a faster method for finding the area (related to halving the area of the rectangle that enclosed the triangle) but he did not explain his method to Pepe whose intense interest in the construction process is described by Leon: "um ... I'm about to do something to it and he ... grabs it off me. But he actually starts doing *work* ... on it". Pepe enlisted Leon's co-operation twice during triangle construction (to hold the end of the ruler as pivot and to read a number from the board). Pepe's exclamation of satisfaction when the construction was complete and his comment "Mrs Milano, I am finished. What do you want me to do now?" confirm Pepe's area of task focus.

As students worked to find the area of their triangle, Mrs Milano moved around the room posing questions (to individual pairs of students). Leon spent most of his lesson time (during Situation 2) leaning from one group of students to another taking part in and frequently instigating off-task talk. He also directed occasional comments and questions (about the mathematics) to Mrs Milano. The quality of Leon's questions to Mrs Milano belied the lack of task focus suggested by the video evidence. Leon considered a variety of questions (Table 2) and synthesised the following ideas (to suddenly grasp reasons for why the area of a triangle formula works): (a) the sequence of problems over the past few lessons; (b) his thoughts over-night; and (c) the working evident in other students' books; (student interview). The pleasure that accompanied this conceptual leap was evident in Leon's exclamation (in class) and his reconstruction of this incident in his interview:

I sort of just remembered what happened 'cause I was thinking about it yesterday and I- I realised that the one [triangle] that had a set [pause] like it was s- straight along one side not angled in [pause]um I- I thought about that that it would be a rectangle and [pause] then the ones today I was sort of looking at them and then I just realised, like I [pause] sort of just in my head I pulled it apart and put them together so that they equalled the same [pause] And it just sort- I just sorta went

'oh' and answered the question ... sorta like [pause] oh wow [pause] It just sorta clicks into your head and and you think oh! I know this now".

Leon's question "if a square is a special kind of rectangle then can't a rectangle be a special kind of square" was unexpected and Mrs Milano provided logical arguments. This was one of the rare occasions (during a non-routine task) when Mrs Milano chose to instruct through transmission rather than initiation of student exploration. Classroom observation supported by teacher interview data indicated Mrs Milano chose this path of action because: (a) the question was unexpected and an investigative pathway had not previously been considered; (b) the time restrictions in a lesson limited the number of individual misconceptions that could be immediately dislodged; and (c) Mrs Milano expected her clear logical argument would be sufficient to dislodge Leon's misconception. Although Pepe understood the logic (explained in Pepe's interview) he did not explain to Leon but quietly muttered "where did you get that shit from?". Leon's reconstructed thoughts about Mrs Milano's logical argument demonstrate its lack usefulness to him:

... she was doing a little check list of all the things that she was doing all the things a square needed to be a rectangle and [pause] they all ticked off and I thought well if a square can have all the same attributes of it as a rectangle, a rectangle has only got one less than a square, so can't a rectangle be a special kind of square? It's almost exactly the same, it's only a little bit different.

*Summary.* In each of the identified situations where concept creation occurred, the student or students selected and explored an idiosyncratic focus within or peripheral to a non-routine task set by the teacher. The mathematics undertaken in the exploration was unfamiliar to the student or students working with the focus question. In the case where a student focused question did not result in conceptual change, student exploration did not occur (Table 2). Where one student possessed conceptual understanding that would answer the focus question of the other student, no more than a cursory attempt was made to share this greater conceptual understanding and the pair chose to work individually.

## Conclusions and Limitations and Implications

Analysis of student activity through the window of *discovered complexity* has been found useful both in this present study of concept creation by Year 8 students of mathematics and my previous study of senior secondary students. On initial exploration, the frequency of concept creation appears higher in this Year 8 classroom than in other Year 8 classrooms I have observed so far during the Learners' Perspective Study. Similar characteristics were identified in situations that triggered concept creation in both the Year 8 study and my previous Year 12 study (Williams, 2000). These characteristics included the use of tasks structured with the opportunity for students working at different levels of sophistication to discover complexities (specifics or generalities; Leon and Pepe in Situation 2); and the use of pedagogical approaches in which (a) student autonomy was encouraged (explore the task in idiosyncratic ways); (b) the *expert other* (teacher) began to 'sets up' the learning situation prior to student commencement of the task (Leon's description of sequence of examples in Situation 2), (c) the teacher is not continually present as the student works with the task, and (d) the teacher asks questions to encourage student clarification, analysis and synthesis (Mrs Milano in Situation 1) rather than providing hints, answering questions, and indicating the correctness of pathways; (e) the

teacher initiates student exploration when a question is asked rather than reverting to a transmissive teaching mode. Student possession of similar speeds of working and similar rates of understanding is not sufficient to ensure collaboration will occur (Situation 2); it appears the students' zones of proximal development must overlap if sustained collaborative concept creation is to occur (present study and Williams, 2000). Overlap in zones of proximal development relies upon similar levels of conceptual development in the domain of the mathematics required to resolve the students' question.

Areas for further research include: (a) how frequently concept creation generally occurs in Year 8 classrooms; (b) whether collaborative concept creation results in a greater conceptual leap than the conceptual leap during individual concept creation; and (c) what factors influenced the extended intervals of sustained engagement in the Year 12 class (Williams, 2000) compared to the shorter intervals of sustained engagement in the Year 8 class. This study identified some but not necessarily all situations that led to the creation of concepts new to these two students. Supplementing the video data (observations of overt classroom behaviour) with student interviews (student reconstruction of their learning during the video stimulated interview) reduced the number of unidentified instances of concept creation. Although this study explores concept creation for only one pair of students, it begins to characterise situations that could lead to concept creation and this informs my intended extended study of these characteristics using a more extensive selection of data from the Learners' Perspective Study. Not only does the present analysis highlight the complex issues involved in providing and sustaining a classroom environment that facilitates creative conceptual change, it also identifies the engagement, perseverance, and feelings of satisfaction and pleasure that can accompany this creation. These findings inform current research into ways to improve the quality of teaching and learning in mathematics (in the middle years of schooling in particular).

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## References

- Barnes, M. (2000). "Magical Moments" in Mathematics: insights into the process of coming to know. *For the Learning of Mathematics*, 20(1), 33-43.
- Clarke, D. (2001). *Perspectives on Practice and Meaning in Mathematics and Science Classrooms*. Dordrecht: Kluwer Academic Publishers.
- Csikszentmihalyi, M., & Csikszentmihalyi, I. S. (1992). *Optimal experience: Psychological studies of flow in consciousness*. Cambridge: Press Syndicate of the University of Cambridge, pp. 251-265.
- Holton, D. & Thomas, G. (2001). Mathematical interactions and their influence on learning. D. J. Clarke [Ed.]. *Perspectives on Practice and Meaning in Mathematics and Science Classrooms*. Dordrecht: Kluwer Academic Publishers, Chapter 5.
- Krutetskii, V. A. (1976). *Psychology of Mathematical Abilities in Schoolchildren*. J. Kilpatrick & I. Wirzup [Eds.]. Teller, J. [Translator.]. Chicago: The University of Chicago Press.
- Reeve, R. & Reynolds, F. (2001). The role of gesture in co-constructing mathematical understanding. D. J. Clarke [Ed.]. *Perspectives on Practice and Meaning in Mathematics and Science Classrooms*. Dordrecht: Kluwer Academic Publishers, Chapter 10.
- Schaffer, H. R. (1996). Joint involvement episodes as context for development. *An Introduction to Vygotsky*. H. Daniels. New York: Routledge, pp. 251-280.
- Thornton, S. (1999). Creating the conditions for cognitive change: the interaction between task structures and specific strategies. *Child Development* 70(3), 588-603.

- Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes*. Cambridge, Mass.: Harvard University Press.
- Williams, G. (2000). Collaborative problem solving and discovered complexity. J. Bana & A. Chapman. [Eds.]. *Mathematics Education Beyond 2000*. (Proceedings of the 23<sup>rd</sup> annual conference of the Mathematics Education Research Group of Australasia, Fremantle, pp. 656-663) Sydney: MERGA.