Williams, G. (2001). Year 8 Maths—Tasks that contributed to student learning. In Colleen Vale [Ed.]. Brunswick, Victoria. Mathematical Association of Victoria

YEAR 8 MATHS—TASKS THAT CONTRIBUTED TO STUDENT LEARNING[1]

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My preliminary analysis of the mathematics classrooms of several Melbourne Year 8 mathematics teachers seen by their school communities to display 'good teaching practices' is reported. This data forms part of a broad international study of the learning of mathematics as viewed from the perspective of the learner. Brief descriptions of the schools, and students are provided along with some of the students' comments about why they found the learning of Mathematics more enjoyable and easier to understand in the classrooms of these teachers. A summary of one teacher's use of several tasks and the subsequent student responses has been described. These tasks were selected for analysis and discussion because there was evidence of student autonomy, spontaneity, and creativity in student responses to these tasks and the students studied displayed high levels of interest, perseverance, and sometimes even excitement as they worked with the tasks. I conclude with my reflections about how this preliminary analysis may assist teachers as they work to make the teaching and learning of Mathematics a more enjoyable and productive experience.

Introduction

This paper recounts my preliminary analysis of students' perceptions about how their Year 8 maths teachers were able to promote enjoyment in the learning of Mathematics and how their teachers found ways to introduce new mathematical ideas in ways that made it easier for the students to understand. A selection of student responses to several tasks utilized by one teacher form part of the analysis of situations that led to student autonomy and spontaneity in the learning of Mathematics. It has previously been demonstrated that student autonomy (Schiefele & Csikszentmihalyi, 1995; Williams, 2000a) and student spontaneity (Williams, 2000a) in the production of new mathematical ideas can be a pleasurable experience. The present study supports this finding and helps to identify ways in which teachers can promote such learning.

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The Learners' Perspective Study

Professor David Clarke of the University of Melbourne is the project leader of this study; he liaises with research teams in ten countries—Australia, Germany, Hong Kong, Sweden, Japan, China (mainland), USA, the Philippines, South Africa, and Israel. Each of these teams is presently researching the teaching and learning occurring in the classrooms of three Year 8 mathematics teachers working in co-educational government settings who are seen by their school communities to display 'good mathematics teaching practice'. The study involves the use of three video cameras in each Year 8 Mathematics classroom for each lesson over a three-week period. The cameras focus respectively on the teacher, the class as a whole and a group of focus students. A different group of students become the focus for each lesson. After the lesson, two focus students are interviewed (generally individually) to gain an understanding of how the students perceived the mathematics lesson and to identify any new learning that occurred for the individual student. These interviews are stimulated by the use of mixed-video-images of the mathematics class the students have just attended with the focus students as center frame and the teacher as an insert in the corner of the image.

Students were asked a variety of questions including: What helped you learn? Which parts of the lesson were important to you and what were you thinking and feeling at those times? What is a good lesson for you? Was this a typical lesson? and, Do you enjoy Maths and maths classes (a lot, a little, in between, or what)? The teacher also took part in a video-stimulated interview once a week. A four to five lesson familiarization period enabled us to gauge the extent to which the class was behaving normally (by considering the students' and teachers' comments in the interviews). Generally, after about four lessons the classroom began to settle back almost to normal and almost forget the three cameras and four researchers perched in every available space around the perimeter of the classroom. Student comments like 'the class is quieter than normal' became less frequent and the teacher looked in our direction less frequently when confronted with those unexpected occurrences that always seem to be part of the every day life of a teacher. Each teacher commented on how useful discussions with the research team had been as an impetus for additional reflection about their teaching practice, and the luxury—not generally available in schools—of having the opportunity to focus on teaching practice and mathematical ideas over an extended period of time.

The schools, teachers and the students' perspective

In addition to the previously mentioned criteria, the teachers were selected so the classes studied included a variety of socioeconomic and geographical areas and a variety of the types of the cultural mix likely to be found in the particular city. The following discussion includes descriptions of the classes and the schools in which the three teachers from Melbourne taught.

Schools

A school from the south-eastern, western suburbs and outer-eastern suburbs formed part of the study. Although all of the schools included some students who came from lower socio-economic backgrounds where financial hardship was experienced, the proportion of these students varied from school to school. Two schools had at least eight different cultures represented amongst the students but the amount of time the majority of the families had lived in Australia tended to differ from school to school. In the western suburbs school, the students were more likely to have arrived in Australia in late primary school or early secondary school. In the south-eastern suburbs school, the students from families who were the more recent arrivals in Australia tended to have come to Australia before Year 8 child was born or at least before the child had reached mid-primary school. In the outer eastern suburbs school, the majority of the students had at least one parent whose family had been in Australia for several generations and the same degree of cultural mix evident in the other two classrooms was not apparent in this classroom.

Teachers

Mrs Milano taught in the south-eastern suburbs school, Mrs Greeno in the western suburbs school and Mr B in the outer eastern suburbs school. These teachers shared a variety of characteristics. Each had developed a classroom atmosphere where the teacher and the students demonstrated mutual respect so instances where a student 'put down' another student—in the classroom—were rare. All three teachers displayed high levels of energy as they taught but each displayed this energy in a different way. Each of the teachers was helpful and approachable to students and frequently moved around the classroom spending time assisting individual students or groups of students. I now describe some of the characteristics of each teacher that appear to be an important aspect of their teaching practice. Identification of these features resulted from student report, teacher recognition, and my own analysis of classroom operation. The level of agreement of ideas drawn from each of these three sources was high.

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Mrs Milano was seen by the students as approachable, 'down to our level', 'always finding interesting things to tell us about', and 'cares whether we understand'. Almost every student commented on how much they enjoyed being in Mrs Milano's class and how much they appreciated the way she taught Mathematics. Mrs Milano had always undertaken further tertiary study each year even though she had been teaching for more than six years. She identified what she learnt through this extra study, and the ideas of several teachers she had worked with as the main influences on her teaching approach. Throughout each lesson Mrs Milano continuously discussed ideas with the class as a whole, groups of students, or with individual students. Where students indicated an interest in further discussion this continued after the lesson where time permitted. Mrs Milano's teaching approach was influenced by her belief that students should be questioned to understanding rather than just told everything. For this reason, much of her 'talk' was in the form of questions. She praised students for their attempted responses even when these responses were incomplete or not quite right. Comments from several students (in their interviews) indicated they valued this approach; it helped them feel comfortable enough to try to answer other questions. Many students commented that one of the reasons they would attempt to answer questions (even when they were not sure of the answer) was because this led to the student finding out whether their thinking was correct or not and assisted in knowing the right answer for next time. Mrs Milano's clear board work, frequent use of clear diagrams, tendency to review the main features of the previous lesson at the start of the next lesson and her clear explanations were other aspects of her teaching approach highly valued by her students.

Mrs Greeno was efficient and effective in the manner in which she monitored and managed students in her classroom. She was 'matter of fact' but also approachable as demonstrated by the number of students who requested extra assistance both in and out of class. Mrs Greeno moved around the classroom helping each student for short periods of time. At almost any instant in time she appeared to be aware of what every student was doing—both with regard to behaviour and with regard to student work. She would make the occasional quiet quick disciplinary comment to a student across the other side of the room (to draw their focus back to their work), and almost simultaneously ask a student in another part of the room about an aspect of the problem that student had been struggling with several minutes before. This helped her select her pathway of progress around the room because sometimes she was able to help a student with a comment directed across the room and at other times needed to return to a student who still struggled after this 'remote control' assistance. This class had a high degree of absenteeism amongst at least 25% of the class members and there were also four to five students who required varying degrees of integration assistance. A stream of different integration aides spent varying amounts of time in the classroom and one student regularly started in the maths classroom but left for extra assistance after several minutes. All these changes in class composition and classroom activities did not appear to impede Mrs Greeno's progress. She provided varying degrees of assistance to different students and appeared to know the present level of progress of each student who had been absent and each integration student with no integration aide present that lesson. Each student in the class had a written contract that contained a list of the work required for that week. Mrs Greeno had a check-list from which she was able to work out who 'owed' her work. Where work was not presented on time, Mrs Greeno—with a minimum of fuss—arranged to speak to the student after class or after school. Mrs Greeno tried to be readily available to provide out of class assistance to any student requiring extra help and students availed themselves of this opportunity. At one stage I was awaiting the arrival of a student (during the research period) when two senior students from the school asked about the video equipment. My explanation that we were researching good teaching practice in Mrs Greeno's classroom led immediately to the response 'oh she's great, you will get no complaint there—she taught both of us in junior years'.

Mr B was always 'Mr B' to his students. In recent years, he had taught only senior classes but this year he had decided to move to teaching Year 8. This change was motivated mainly by the school's decision to stream their two Year 8 maths classes. Mr B was concerned that those students in the lower stream would feel undervalued. To diffuse this situation he elected to take this class and continually worked to help students recognize their worth. Many students in Mr B's class commented on how much easier it was to understand maths in his class. The substance of two comments from a self-willed student who could have been difficult to manage in some classrooms captures the essence of why the students felt they achieved well in Mr B's class:

> He's not like other teachers who just go 'bla bla bla' until they have said it all … and then stop. He actually stops and looks to see if you understand and if you don't he explains it another way.

> He treats you like a real person and he acts like himself not like a teacher … here … look at this [student flicks remote control and finds the part where the teacher apologises for being a little late 'because my daughter fell off the couch onto her head just as I was leaving']. See, he apologises to *us* … he treats us like real people.

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Students said they liked Mr Bs class because you never knew what was going to happen next—one day he could be standing on the desk top as Captain Compass waving a compass in his hand ready to teach a lesson on construction, another day he could be licking the back of a piece of paper because he wanted to quickly attach it to the board to demonstrate the angles in a triangle add to one hundred and eighty degrees, and on yet another day he could be praising the class for how far they have come with being responsible for themselves whilst simultaneously urging them to take the next step in developing further responsibility. 'He has shown us we can do things for ourselves, he had faith that we could do it'—were the types of comments from the two boys interviewed that day. Mr B used a similar emphasis on development of student meta-cognition in aspects of his teaching; he discussed the value of mistakes 'because we learn so much from them', suggested students look for the common types of mistakes if a problem was not working, and encouraged students to discuss the ways they might be able to check their working.

The students' perspectives of the lessons and teachers

The majority of the students thought a good lesson was a lesson in which they learnt something new although the occasional student identified a good lesson as one in which they did not have to do much work. Many students mentioned how much more they enjoyed mathematics this year than in most previous years. Students attributed their enhanced enjoyment to such reasons as: the teacher: 'doesn't yell at you when you get things wrong', 'will explain again when you don't understand', 'tries to make the lessons fun', 'makes sure there is time for everyone to have their turn [individual time with the teacher]', 'uses language that we understand', and 'uses explanations that are easy to understand'.

One very powerful message from students [in all three schools] was how important it was [to the student] to understand the mathematics they encountered in the classroom. Many students spontaneously took the opportunity to use the video replay to go over things they had not understood in class and to extend their understanding of these ideas during the interview. This was not an intentional part of the interview design but occurred frequently because the student was in charge of the remote control and perceived those parts of the lesson they had not fully understood as important. The strength of emotions associated with success or lack of success in developing an understanding of mathematical ideas was displayed in various ways by a large number of students in the interviews.

The tasks, the teaching and the responses of Leon and Pepe

The two students (Leon and Pepe) were selected for this preliminary analysis because of the different ways they responded to several tasks throughout the three-week period. Where most of the time these students participated in the mathematical activities undertaken by the class and simultaneously participated in varying degrees of off-task talk, on the occasions studied here, Leon and Pepe were more autonomous and spontaneous than usual in the way they approached several mathematical ideas.

Upon initial inspection, there may not appear to be anything very special about any of the tasks the students undertook on these occasions—these tasks appear similar to numerous tasks used in Year 8 classrooms in many countries including Australia. What was particularly interesting in these cases was that the teacher did not bring the tasks to fast closure, and the teacher demonstrated she valued autonomous student exploration of mathematical ideas whether they were ideas the teacher had intended to develop or ideas spontaneously explored by the students.

Task 1

Students had worked with the concept of perimeter in the lesson before Task 1 was introduced and in that lesson they had found the perimeter of a variety of simple shapes. To introduce Task 1, Mrs Milano drew a rectangle on the board, labeled **l** for the length and **w** for the width, placed **P = 38 cm** beside the diagram and asked:

How many rectangles have a 38 cm perimeter?

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TEACHER IMPLEMENTATION OF TASK 1

Mrs Milano assisted the class to develop ideas by writing student suggestions of specific examples in a table on the board and altering these examples in response to class consensus about necessary changes. Once all the integer values for the dimensions of the rectangle had been supplied, Mrs Milano continued to ask for further responses stating several times: "the only restriction is that the perimeter has to be 38 cm". Mrs Milano's questions and the comments in conjunction with other comments from other class members led to the rejection of suggestions of negative numbers and zero—because they lacked meaning in relation to a rectangle. When all student responses appeared to be exhausted, Mrs Milano commented: "I am suggesting that there are other solutions". This precipitated student suggestions of decimal dimensions and this resulted in some students recognising and articulating possible patterns and Mrs Milano asking why these patterns existed. The discussion continued until the class began to recognise an infinite number of solutions were possible. At this stage, Mrs Milano began to introduce the next task. Mrs Milano's questions during Task 1, and her point of task closure indicated she intended students to recognise: (a) a general formula **l + w =19**; (b) a reason why this formula works; and (c) that an infinite number of rectangles were possible.

LEON'S, AND PEPE'S RESPONSES AND INTERACTIONS WITH MRS MILANO AFTER TASK 1 CLOSURE

As Task 1 finished and Mrs Milano began to prepare the students for the next task, Leon and Pepe simultaneously and collaboratively began to explore their own spontaneous extension to Task 1—is there a relationship between the number of possible perimeters where you include numbers written to different numbers of decimal places in separate categories? Although they were not the focus students for that lesson, Leon and Pepe's interest in their own development of new mathematical ideas could be seen and heard across the classroom—evidence was drawn from my field notes, the video data, and student interviews (two weeks later). Several minutes after Leon and Pepe displayed this high level of interest, they began to ask questions and make comments to Mrs Milano. Both students appeared to be building ideas together in the few minutes before Leon asked Mrs Milano: "Do you know the answer if there was only one decimal place or …?" Fragments of Leon and Pepe's interchanges with Mrs Milano indicated the students' development of their newly forming ideas in every-day language (Reeves & Reynolds, 2001): Leon "Nine hundred and ninety". Pepe "Nine thousand nine hundred and ninety" [simultaneously]; Leon "and so on and so on and so on"; and Pepe "it just keeps on going and going and going". Leon recalls some of the generalities of the pair's creative development of new mathematical ideas (in his interview two weeks later):

> ... and then if you go to two decimal places you can only have so many different decimal places for that number and like it kept building and building and building and you could do it … um you sort of found a formula for it … a way to ... categorise it into like different numbers

During the exchange with Pepe and Leon after Task 1, Mrs Milano simultaneously continued to organise the whole class for the next task (dealing out string; requesting a student draw a circle on the board; responding to student questions, praising the student's circle drawing; and recalling an historical anecdote about a well known Italian drawing a circle). Examples of Mrs Milano's responses to Pepe and Leon included: "Let me think about it. Can I think about it and you can as well?" "So you've got two decimal places. Why plus 90? ... Oh you've got the single decimal place plus the whole number?" "Well, you tell me. Would there? Why?"; "Yes there is an answer and I just can't think …" "It depends too ... that's a really interesting question. Is 10 and 9 different from ten point zero and nine point zero and is that different from ten point zero zero and nine point zero zero?" Even though, at the time, Mrs Milano did not know the solution to the problem posed, her behaviour was consistent with her teaching philosophy (as explained in the teacher interview) and consistent with her usual behaviour to encourage students to develop ideas for themselves. The extent of Leon's and Pepe's interest and engagement in the autonomous and spontaneous development of the new mathematical idea is evidenced by: (a) Leon and Pepe's high level of engagement in the time just prior to their interaction with Mrs Milano; (b) the joint and sustained contribution of both students to the discussion with Mrs Milano (even though the next class task had commenced); (c) the exclamations as each student recalled this episode in their interviews two weeks later, and (d) Leon's description of this and other collaborative experiences:

> Um Pepe and I are sort of ... at the same ... level of intelligence as well as like level of ... you know friendship and stuff ... that's why we work really really well together. 'Cause n- it's not just one of us doing all of the work … and one of us just getting all of the answers and the other one sort of thinking 'oh what's going on here?' ... it's almost like we combine ... what we are thinking about and we sort of come out with the same answer.

Leon explained the *same level of intelligence* to mean to work at the same pace and to understand new ideas at the same rate. Leon thought he and Pepe had different levels of intelligence in other subjects. Leon's perspective on how his collaboration with Pepe works could inform our use of grouping strategies during normal classroom practice.

Task 2

Mrs Milano drew three triangles of different shapes on the board (one right angled triangle, one scalene triangle with three acute angles and one scalene triangle with one obtuse angle). She then allocated each student pair a particular triangle and asked them to find the area.

TEACHER IMPLEMENTATION OF TASK 2

Although the area of a triangle as equal to half the base by the height had arisen informally in the previous lesson (in a case where a right angled triangle was displayed as half of a rectangle), the formula for the area of a triangle had not been formally taught to these students during their Year 8 mathematics course at the time this task was undertaken. There is no doubt that most students would have encountered this formula for the area of a triangle in previous years but the approach of using a formula to find the area was not the approach focused on by most students. Although no explicit process to solve the task was given, the intention of the teacher had been for students to draw upon the work they had undertaken in the previous lesson and count squares to find the area. In the previous lesson, students had drawn the outline of their own hand, superimposed a grid, and worked out an estimate of the area within their hand outline by counting squares. As students worked to find the area of their triangle, Mrs Milano moved around the room posing questions (to pairs of students) and responding (sometimes with private-talk and sometimes with public-talk) to students' questions.

PEPE'S (AND LEON'S) RESPONSE TO TASK 2

Leon and Pepe failed to reach consensus about the level of generality with which they would approach the teacher's task so the students chose to work individually formulating their own idiosyncratic foci. Leon recognised a faster method for finding the area (related to halving the area of the rectangle that enclosed the triangle) but he did not explain his method to Pepe whose intense interest in the construction process was described by Leon: in his interview "um ... I'm about to do something to it and he ... grabs it off me. But he actually starts doing *work* ... on it". Pepe was determined to use his own method (constructing the triangle and counting the squares) to find the area of the triangle. Pepe leant over his page and displayed a perseverance and intensity as he thought about and constructed a triangle using the dimensions shown on the blackboard. Over a period of time, Pepe remained completely engrossed in his construction with his eyes focused on his page except for several short time intervals when he either quickly (and in staccato fashion) demanded the use of construction equipment belonging to students around him or demanded the short term co-operation of Leon in to perform of certain manual tasks required in the completion Pepe's construction. In his interview, Pepe confirmed that the construction of a triangle when three sides are known was unfamiliar to him. He explained that he firstly realized he could use a compass to construct the triangle but when he found the compass would not extend the required distance he thought about using a ruler and enlisting Leon's help to keep the pivot in place as he made an arc. Pepe enlisted Leon's co-operation twice during triangle construction (to hold the end of the ruler as pivot and to go out and read a number from the board). Pepe's exclamation of satisfaction when the construction was complete and his comment "Mrs Milano, I am finished. What do you want me to do now?" confirm Pepe's area of task focus had become the construction of the triangle rather than the finding of the area of the triangle.

Task 3

Task 3 was a series of tasks over three consecutive lessons. In the first lesson, students counted squares to find the area that lay within the outline of their hand, and worked out the area of a right-angled triangle drawn so the hypotenuse of the triangle was the diagonal of rectangle superimposed upon the triangle. The second lesson included the task described as Task 2, and the third lesson focused on whole class discussion of student solutions to Task 2.

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LEON'S RESPONSE TO A SEQUENCE OF TASKS PRESENTED OVER THREE LESSONS (TASK 3)

In the first of the three lessons the sequence of tasks spanned (referred to as Task 3), Leon (and Pepe) participated in the classroom activities with some simultaneous off-task talk. As described under Task 2, by the time Leon and Pepe began to work on Task 2, Leon had already developed an interest in ideas focused around why the area of a triangle would always be equal to half the base by the height. From lesson one in the sequence he had realized why this was so for a right-angled triangle but he was still puzzling over why it was so for other triangles. He was not interested in following Pepe's pathway of constructing triangles and counting squares to find the area because he thought there was an easier way but was still trying to crystalise his ideas. In the lesson where Task 2 was undertaken, Leon's behaviour differed markedly to Pepe's. Where Pepe displayed an intense interest in his own triangle construction, Leon spent most of his time during this lesson leaning from one group of students to another and taking part in and frequently instigating off-task talk. Leon's behaviour (considered in isolation from his after-class interview) would have suggested Leon did almost nothing other than engage in off-task talk with students sitting around him. These off-task interactions were sometimes intended to be provocative as evidenced by a question to one of the girls about why she hit her friend when in Grade 3. Leon also directed occasional comments and questions (about the mathematics) to Mrs Milano. The quality of Leon's questions to Mrs Milano belied the lack of task focus suggested by the video evidence. Some of his questions and comments related to this idea he was puzzling about (why the formula for the area of a triangle works). The substance of these questions and comments included: 'Are any of those triangles [on the board] easier to find the area of than others?'. Leon also asked several questions about other concepts he was puzzling over but these questions and ideas are not reported here.

It was only when Leon's occasional questions to the teacher were considered in conjunction with his reconstruction of his thoughts during the lesson, his thoughts over the intervening nights, and his explanation of the exclamation that burst from him in class during the third lesson, that the depth of Leon's thinking becomes clearer. Over a three-day period, Leon had autonomously developed an understanding of why the area of a triangle is always equal to half the base by the height. To achieve this end, Leon considered a variety of questions and synthesised the following ideas to suddenly grasp the new insight. These sources of assistance included: (a) the sequence of tasks over lesson one two and three of this task sequence; (b) his thoughts over-night; and (c) the working evident in other students' books while he was leaning around looking at everything in lesson two, and the process of going over solutions. The pleasure that accompanied Leon's final conceptual leap was evident in Leon's exclamation (in class) and his interview reconstruction of the incident:

> I sort of just remembered what happened 'cause I was thinking about it yesterday [lesson two of the sequence] and I- I realised that the one [triangle] that had a set … like it was s- straight along one side not angled in [right angled triangle] ... um I- I thought about that that it would be a rectangle and ... then the ones today I was sort of looking at them and then I just realised, like I ... sort of just in my head I pulled it apart and put them together so that they equaled the same ... And it just sort- I just sorta went '*oh*' and answered the question … sorta like ... oh wow! ... It just sorta clicks into your head and and you think oh! I know this now".

TEACHER IMPLEMENTATION OF TASK 3

Mrs Milano's contribution to the development of new mathematical ideas by Leon in this case is not as clearly visible. Mrs Milano was busy with other students (and some of Leon's other questions) so did not respond directly to Leon's questions and comments about which triangle was easier to find the area of. The public-talk in the classroom related to finding the area, and not to the reasons why a formula for the area of a triangle worked. On closer inspection, Mrs Milano's contribution to Leon's development of a new idea was substantial. She sequenced tasks over three lessons and this provided support for Leon's thoughts about what was happening mathematically. By spreading these tasks over three lessons, she provided time for Leon to puzzle over these ideas at home. By not just giving the reasons in the first lesson, she left unanswered *but accessible* ideas to pursue. In addition, her normal classroom practice (which encouraged students to think for themselves) could have contributed to Leon's inclination to spontaneous generate his question.

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Summary and conclusions

In each of the identified situations where students autonomously developed a new mathematical understanding, the student or students selected and explored an idiosyncratic focus within or peripheral to a non-routine task set by the teacher. The mathematics undertaken in the exploration was unfamiliar to the student or students working with the focus question and as the task was not brought to fast closure, there was time for student exploration. This type of student exploration (discovery of complexities) was also identified in my study of Year 12 Specialist Mathematics students (Williams, 2000a). Similar student responses have also been identified with students working in collaborative groups on unfamiliar challenging problems at year levels ranging from Year 4 to Year 12 (Barnes, 2000; Williams 2000b; Williams, Peck et al., 2000). These students also spontaneously focused their own questions and used mathematical ideas new to all group members in the resolution of the focus questions to 'discovered complexities'.

It appears the opportunity for several consecutive discoveries of complexities may not be provided by Task 1 and 2 in the present study although Pepe's period of intense perseverance in Task 2 could be viewed as the successive discovery of two complexities: 'how can I make this triangle?' 'How can I make it when the compass won't reach?' Leon's maintained interest in Task 3 over several days suggests that the progressive provision of new supporting tasks over several days led to the sequential discovery new complexities like 'Are some triangles easier?' on day two. Leon's use of the mathematical thoughts of others (work in other students' book works) as part of a synthesis of his own ideas suggests he could benefit from participation in collaborative group work (Barnes 2000; Williams 2000a; Williams, Peck et al. 2000) which could increase Leon's focus on justification and the oral communication of mathematical ideas which could in turn facilitate further synthesis. This would be consistent with the learning strategies Leon possesses presently

The mathematics classroom in which Leon and Pepe participated has provided many learning opportunities. Some of the tasks have enabled students to work at different levels of sophistication (Leon working in generalities and Pepe working in specifics as they considered Task 2). Students were able to explore tasks in an idiosyncratic manner (Leon and Pepe in Task 1 and 3) and student thinking was supported by the task sequence provided (Leon with Task 3). The teacher facilitated student development of ideas by the open questions she generally asked (Mrs Milano in Task 1) in preference to providing hints or answering questions directly.

Student possession of similar speeds of working and similar rates of understanding was not found sufficient to ensure collaboration would occur (Leon and Pepe in Task 2); collaboration only occurred when the mathematical ideas were new to both students (Task 1). This finding suggests the popularity of peer tutoring as a teaching approach may need review.

Not only does the present analysis highlight the complex issues involved in providing and sustaining a classroom environment that facilitates autonomous, spontaneous and creative development of mathematical ideas by students, it also identifies the engagement, perseverance, and feelings of satisfaction and pleasure that can accompany this creative process. This study provides some insights into the ways teachers can sequence tasks, and utilise short open-ended tasks as they work to develop autonomous mathematical thinking by their students. The fact that students in the present study have been found to discover some complexities but not the large numbers of successive complexities evident sometimes with the use of extended open-ended tasks suggests exploration of the use of extended open-ended tasks in collaborative settings could be a productive extension to these ideas.

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