STUDENT INCLINATION TO WORK WITH UNFAMILIAR CHALLENGING PROBLEMS: THE ROLE OF RESILIENCE[1]

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My research shows links between student resilience (ability to bounce back after failures, or after exposure to stressful situations), and student inclination to explore unfamiliar mathematical ideas. Students' oral responses can provide indicators of presence or absence of resilience, and there are actions teachers can take to increase the resiliency of students in their classrooms. Interview responses, from Year 8 students in Australia and the USA, are used to illustrate how teachers could 'read' indicators of resilience. Illustrations from my own teaching approach are then used to suggest how student resiliency can be increased through collaborative problem solving.

Introduction

Almost ten years ago I recognised flow (Csikszentmihalyi, 1992) as a key concept when considering student engagement in mathematics. I discussed this concept in a keynote address at an MAV conference (Williams, 1997). Over the years, I have studied classrooms, including my own, to identify the actions of teachers that support this flow process. I have also studied students, and their inclination to engage in flow situations in mathematics. My friend and colleague Myra Mitchell (a psychologist here in Melbourne) alerted me to the work of Martin Seligman (1995) who found student resilience has the potential to increase emotional well-being. As a result, I recognised resilience as a possible factor associated with student inclination to engage in exploring unfamiliar mathematical ideas (where the process of problem solving often involves 'many failures before a success'). On reflection I could see that my classroom environment included many of the characteristics likely to build resilience. I decided to research this student characteristic as part of my study of factors that promote or inhibit student exploration of new mathematical ideas. You can imagine my excitement when I found Martin Seligman described the building of resilience using the flow model. In the following sections, I address three questions: How can we recognising resilience or lack thereof? What is flow, and how is the state of flow associated with building resilience? And, how do these ideas relate directly to the teaching and learning of mathematics?

How Can We Recognise Resilience or Lack Thereof?

Resilience relates to how a child explains occurrences in their day-to-day encounters with the world. This explanatory style influences the child's orientation to the world (Seligman, 1995); a child may have an optimistic orientation to the world (be resilient), a pessimistic orientation to the world, or an orientation between these two extremes.

Optimistic children perceive good fortune to result from their own endeavours rather than occur as a matter of chance; they see failures as temporary and as able to be overcome through persistence. They also generalise successes as personal attributes and constrain failures to the specific situations in which the failures occurred. Table 1 summarises Seligman's dimensions of optimism; used as indicators of resilience in my research. The student interview excerpts (from Year 8 mathematics students in Australia and the USA) were part of individual video-stimulated interviews after mathematics lessons. The student controlled the remote and fast-forwarded through the lesson to find and discuss the parts of the lesson that were important to that student (for any reason).

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[1] Williams, G. (2003). *Student inclination to work with unfamiliar challenging problems: the role of resilience*. In Barbara Clarke, Alan Bishop, Rod Cameron, Helen Forgasz, Wee Tiong Seah. Making Mathematicians. Brunswick, Victoria: Mathematical Association of Victoria.

The students whose comments are used to illustrate resilience, and lack of resilience along each of the three dimensions of optimism include Leon, Pepe, Eden, Dean, and Kerri (who demonstrated an inclination to spontaneously explore new mathematical ideas), Darius (who persistently used trial and error to get answers, but did not think about the mathematical meaning), and Lara, Sally, and Jason who were not identified spontaneously exploring new mathematical ideas during the research period, and whose ways of working mathematically suggested they would be unlikely to do so at that time. I selected these students for my illustrations to provide excerpts that were rich enough to show indicators at both ends of the spectrum for each dimension (see Table 1).

Table 1. Seligman's (1995) three dimensions of optimism used as indicators of resilience

Dimension	Indicators of Lack of Resilience		Indicators of Resilience	
	Success	*Failure*	*Success*	*Failure*
Permanent-Temporary	Temporary	Permanent	Permanent	Temporary
Pervasive-Specific	Specific	Generalised	Generalised	Specific
Personal-External	Attributed to external factors	Attributed to personal factors	Attributed to personal factors	Attributed to external factors

Of the students who spontaneously created novel mathematical concepts (Kerri, Leon, Pepe, and Eden) or developed novel mathematical ideas (Dean), Kerri and Leon both achieved excellent grades in mathematics, Pepe achieved above average grades, Eden gained average grades in the year of the study (he also achieved 13 for problem solving on a national competition where the average Year 8 score was 3), and Dean struggled to achieve average grades. Of the students who did not demonstrate an inclination to explore new mathematical ideas during the research period, Darius, Lara and Sally achieved high grades, and Jason (who focused on off-task interactions with his peers rather than his work) achieved low grades. Sally was task-centred and Lara ego-centred (Nicholls, 1983). Sally worked at a faster pace than most class members and continually relied upon the teacher to check the correctness of her progress or assist when she had difficulties. Lara focused on the effect her mathematical performance had upon other class members (and the teacher) rather than upon the mathematics itself.

Indicators of Resilience

A resilient student is a student who demonstrates indicators of optimism in each of the six different categories listed in the last two columns of Table 1. Each of the three dimensions of optimism contains two indicators; an indicator for contexts in which 'success' has occurred, and an indicator for contexts in which 'failure' has occurred.

Table 2. Evidence of Kerri's resilience using Seligman's (1995) three dimensions of optimism as indicators

Dimension	Indicators of Lack of Resilience		Indicators of Resilience	
	Success	**Failure**	**Success**	**Failure**
Permanent-Temporary			I just catch on quick	Oh, it's still a little bit foggy.
Pervasive-Specific			I: How did you figure that out? K: I don't know. I guess I'm smart	Well, that was a bogus answer.
Personal-External			'[I understand when] I don't need any [more] questions answered '	[I didn't understand because] he [last year's teacher] would try and show us that while we were still learning the … [first idea].

Key: [] square brackets contain words I have added to the student comment to place it in context

Kerri (see Table 2), in her interview, frequently made comments that provided evidence of her optimistic orientation to the world. The table includes an example of a comment that provides an indicator for each of the six categories. Frequently, comments that provide an indicator along the permanent-temporary dimension also contain an indicator along the personal-external dimension; students who see failure as temporary (optimistic on the permanent-temporary dimension) often comment on how they change the situation (optimistic on the personal-external dimension). Kerri has demonstrated she: (a) sees success as permanent and failure as temporary; (b) generalises success and constrains failure to the specific case; and (c) she attributes success to her own efforts and failure to external factors. Kerri demonstrated resilience along all dimensions. She perceives herself as someone 'smart' (generalised or pervasive) who catches on quickly (a permanent characteristic). Kerri determines what she understands by whether she needs to ask more questions. This indicates Kerri sees her success as achieved by her own efforts. When she does not understand, she sees this as a temporary state 'still a little bit foggy'. When Kerri had more difficulty understanding (over a period of time) in Year 7, she identified the teacher's part in her lack of understanding (he taught new things before the original ideas had been taken in). The indicator under pervasive-specific (in a situation of failure) relates to another student but provides an indication of Kerri's orientation; Kerri constrained the 'failure' to the situation 'that was a bogus answer' rather than generalising it to an attribute of the other student.

Comments from other students are now used to illustrate the variety of ways in which students might provide indicators. To 'feel brave' Lara volunteered to answer questions when no one else volunteered. These questions did not involve new mathematical ideas but Lara was uncertain about the answer:

> It is kind of nerve wracking , you're like ohhh 'they're going to laugh at me, hope they don't think I'm stupid'. It sort of makes me feel more *brave*. I think 'hey I'm good for doing this'

This statement of Lara's shows a tendency to generalise from a specific to a pervasive category with respect to successes and failures. Putting her hand up to answer fell into the general category of 'brave' (optimistic on the pervasive specific dimension in terms of success), and getting it wrong was generalised to being seen as 'stupid' (lack of optimism on the pervasive specific dimension in terms of failure). Compare Lara's responses to failure to Eden's response (below).

Eden was a member of a high ability class in which the focus was generally on procedures and answers rather than solution pathways and thinking processes. Although Eden's teacher perceived him to be an average student in mathematics, evidence to the contrary was available (national test results, and Eden's mathematical discussions in the interview). In his interview, when Eden was asked whether he thought he was 'very good at maths, good, about average or what, and how he made that decision', Eden did not generalise his average scores in this class to being 'average at maths'. He qualified his statement about 'being pretty average' to 'in this class' (specific not pervasive with regard to failure):

> Well in this class I think I am pretty much average [pause] because I get pretty average scores [pause] there is no way to explain.

In describing his usual way of collaborating with Pepe, Leon demonstrated he perceived success as permanent and resulting from effort (optimistic along permanent-temporary and personal-specific):

> … we work really really well together 'cause n- it's not just one of us doing all of the work we both always work it out

Darius demonstrated optimism along the permanent-temporary and personal-external dimensions when he described how he approached a problem solving task:

> … keep on trying until you get the system of it

Jason displayed contradictory indicators on the personal-external dimension; he judged himself 'good at maths' using his mother's opinion (rather than by making his own decision). Jason blamed the teacher's short pause time between questions for his perception that he did not appear 'good at maths' in class. Jason has used external sources to judge his success and explain his failure in the mathematics classroom. This provides contradictory information along Seligman's dimensions of optimism and indicates a lack of resilience.

Dean (see Table 3) demonstrated optimism on the personal-external, and permanent-temporary dimensions by his habit of working out what he didn't understand (while the teacher presented familiar work). He illustrated that he perceived success as permanent; by stating 'and then I get it'. Dean recognised his stress when he encounters something new (because he doesn't understand new work easily). In the same sentence he demonstrated that he saw this as a temporary state; because with effort and possibly extra assistance he would work it out. The temporary nature of failure, and the personal efforts by which success is eventually gained can be seen in many of Dean's comments.

Table 3. Evidence of Dean's resilience using Seligman's (1995) three dimensions of optimism as indicators

Dimension	Indicators of Lack of Resilience		Indicators of Resilience	
	Success.	**Failure**	**Success**	**Failure**
Permanent-Temporary			I like lessons that like I don't know what … going on, cause then I- I get it and then I um [pause, 3 secs] and then I write it all down and then I get it- …	'[When I don't know] I will try and do it. If I can't then I'll do it at home or just keep doing it. Then I'll ask my parents or the teacher or person next to me or something. … when I first get a sheet … which I've never done before … I get a bit [pause] stressed [pause] cause the first time I do stuff I always don't get it [pause] at first- it takes me like [pause] a little while- that's why I go over it and over it …
Pervasive-Specific				… I didn't know where the corners went in it. I thought- see when he told me you put the corners facing in I knew what I had to do
Personal-External			[to learn] I write it down in my book and then when he's talking or something that I have already known, then I just look over it again. I thought it up- yeah I thought it up by myself … [the novel idea]	… the first time I do stuff I always don't get it, at first- it takes me like [pause] a little while- that's why I go over it and over it …

Key: [] square brackets contain words I have added to the student comment to place it in context; … part of the dialogue omitted; - self interruption

The lesson for which Dean was the focus student involved the students following a procedure demonstrated by the teacher (Williams, 2002). Students were expected to find the sum of the interior angles of polygons by segmenting each polygon into triangles using straight lines to connect the vertices. By tearing the angles ('corners') off the triangles and placing them together, students were expected to work out the sum. Dean struggled in the lesson because he could not work out how to place angles adjacent to each other. This difficulty occurred because Dean did not recognise angles in polygons (even in triangles). He eventually worked out his own way to solve this problem 'each triangle in the polygon is a hundred and eighty so you need a hundred and eighty for each triangle', and checked his idea using three specific cases to see if it worked.

Dean spent a major part of the lesson not knowing what to do. He requested help from the teacher who came over and demonstrated the process. Dean did not use the teacher's method; he puzzled over the table of results on the board and created his own method. In his interview, Dean did not generalise this lack of initial success to some general attribute of himself. He identified the specific problem; he had not realised that the 'corners had to point in' (optimistic along the pervasive-specific dimension for failure). Although Dean provided no indicators along the pervasive-specific dimension in relation to successes, he did not provide contradictory indicators .

How does knowledge about student resiliency help teachers of mathematics? If one of the characteristics of students who show an inclination to work with unfamiliar mathematical ideas is resilience, and if resilience can be developed, then building resiliency in our students has the potential to improve mathematical performance. Teachers need to explore ways to simultaneously increase student understanding of mathematics and student resilience. The following two section address these ideas by providing: (a) a theory of how to build resilience, and (b) a way to apply this theory to collaborative mathematical problem solving in classrooms.

What is Flow and How Can Flow Help Build Resilience?

Seligman (1995) demonstrated the resilience of a child could change over time. He used Csikszentmihalyi's (1992) concept of flow as a framework to discuss how an optimistic orientation to the world can be developed in young children to increase their resilience. Flow is an optimal learning condition that may occur when a person works just above their present skill level on a challenge almost out of reach. Individuals or groups in flow become so engrossed with the task at hand that they lose awareness of self, time and the world. Seligman found that in overcoming small challenges to gain successes, the child's inclination to undertake future challenges was increased. During my research studies, I have identified the conditions for flow specific to mathematical problem solving (See Figure 1). In this case, the 'skills' are 'mathematical skills and concepts' and the 'challenge' is a student-initiated and student-directed 'intellectual challenge' to explore a mathematical complexity. This involves synthesising presently known ideas and concepts to produce new mathematical concepts whilst progressively evaluating the consistency of the findings (See Figure 1).

Key: ➡The process of discovering complexities in mathematical problem solving tasks

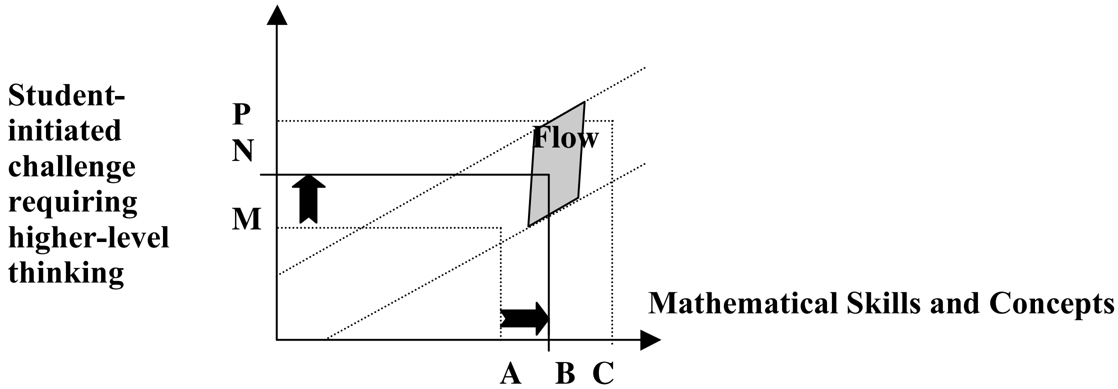

Figure 1. My representation of associations between discovered complexity and flow

A student’s perceived level of skills and concepts and perceived intellectual challenge comfortably overcome are represented by **A** and **M** respectively. By the student focusing their own spontaneous question in relation to mathematical complexities they discover in the task, they can set their own intellectual challenge. Students in flow are seen as working to achieve a goal represented by a point within the shaded region of flow **(B, N)**. They are working just above their perceived skills and concepts level (horizontal black arrow) on an intellectual challenge they set themselves (vertical back arrow). During this process, the group develops a new conceptual understanding about the mathematical complexity they explored. By doing so, they overcome the student-initiated challenge. Each student’s perceived skills and concepts level has then increased to **B** and they can comfortably achieve a challenge of **N**. The shaded region representing flow is then located to the right of **B** above **N** between the parallel lines. To sustain flow would require a discovered complexity that led to a goal represented by a point like (**C, P**). (Examples of ideas students might pursue can be found in the next section). The inclination to work with unfamiliar mathematical ideas is represented on this diagram by simultaneous movement along the two black arrows—using higher level thinking to create novel (to the students) mathematical concepts. Knowing the theory is interesting but unless these ideas can be directly related to student learning in mainstream mathematics classrooms, the potential gain has not been achieved .

How Do These Ideas Relate Directly to Maths in Classrooms?

Building resilience through mathematical problem solving requires attention to a variety of factors including the use of: a) tasks that provide opportunities for students to explore mathematical complexities; (b) a teaching approach that supports independent student exploration; (c) group composition that supports similar paces of working; and (d) a feedback system which gives all class members access to ideas presently being explored in the classroom(Williams, 1996). In the present article I focus on two aspects of my classroom practice that are hidden within 'providing complex tasks' and the 'teaching approach': (a) setting up situations where students can recognise relevant mathematical ideas and bring them into the exploration themselves rather than have the teacher identify these mathematical ideas; and (b) using the reporting process as a way to allow groups to showcase their thinking, and realise that their efforts are contributing to the body of mathematical knowledge the class is building. To discuss these ideas, I use an example task: 'Find a rule or relationship which links the gradient of a line to the angle that the line makes with the positive direction of the **x** axis'. Further detail about ways to present this task (using collaborative group work) can be found in Williams (1996, Investigation 8). I developed this task for Year 10 students because I had found that Year 11 and 12 students continually forgot m = tanø. This task is designed to give students access to a variety of levels of specificity or generality, and a variety of degrees of sophistication.

Setting Up Situations for Recognising

This provides opportunities for students to show mathematical thinking of a quality that is often denied to them in mathematics classrooms where the syllabus is taught in a lockstep manner.

Table 4. Illustrations of ideas and concepts could be recognised and how they could be used (in Investigation 8, above)

Concepts and Ideas Recognised	How the mathematical ideas could be used
Gradient is the slope which is how much the line rises for a run of one unit	Draw a line, measure the angle with a protractor, and find the gradient using rise over run.
Algebra can be used to draw particular lines	The measure of the gradient can be 'read' from the equation
Right-angled triangle; trigonometry; lines can be drawn to go through particular points	To find the exact size of the angle using integer co-ordinates
Pythagorean triangle connects tan functions and gradients	Draw in the triangle and recognise the connection between tan of the angle, and gradient.
Table of tangents of Angles written in increasing order	Recognise table of tangents is approximately the same as table of gradients, and search for possible reasons why.

Table 4 illustrates *some* of the mathematical ideas and concepts students may recognise as relevant, and choose to use in their explorations of the task. With regard to building resilience, these students are moving from a state of what could initially be considered 'failure to see what to do' to a situation in which their own perseverance has helped them discover a way to gain 'success'. Once you begin to think about the different ways groups might approach this task, you can see how regular feedback to the class as a whole could help to interconnect mathematical ideas.

Table 5. Examples of teacher questions to small groups, and some possible small group responses

Teacher question to small groups	Examples of resulting student focus
What could vary and how broadly can it be varied?	Angles, gradients, right-angled triangles, equations, integer values,
Can you see any patterns?	There is always a right-angled triangle; there is more than one right-angled triangle for each graph; the gradient increases as the angle increases; the increase in gradient is bigger each time; the gradient increases as the angle increases up to a certain angle; lines that make an angle of more than 90° with the **x** axis have a negative gradient
Why do *these* patterns apply? What is going on?	The right-angled triangle is formed by a part of the line, the increase in **x,** and the increase in **y,** so there should always be a right-angled triangle there; the negative gradient is because either the **x** or the **y** is decreasing not increasing;
Can you see another way to check your findings?	Connecting the tan**ø** table, and the gradient table by recognising tan**ø** in the right-angled triangle

Are you sure you have varied things as broadly as possible?	Angles greater than 90°; lines sloping the other way; horizontal and vertical lines.

Table 5 includes some of the types of questions the teacher could ask small groups, and some of the ideas groups may focus on as a result. Look carefully at the questions. They do not tell the students what pathways to take or what to focus upon. They are a broad set of questions applicable to many problem situations. By framing questions in such a broad manner, the students are in control of the directions they take. These questions help students to explore further and recognise some of the limitations of their earlier findings.

Using the Reporting Process to Share Mathematical Ideas, and Build Resilience

This task would be undertaken over three to four single lessons or two double lessons. A small group collaboration, class feedback cycle is repeated four times over that interval. Students work in small groups for about ten minutes then each group gives a short (1.5 min) oral report to the class. The cycle is then repeated with another 10-15 minutes work and another set of reports (with different reporters). Groups choose their own focus for the report from the following suggestions:

- something you are not sure about and you are hoping another group can help you with
- something you didn't know before and you do know now
- some pattern you have found
- a mathematical argument for why this pattern exists
- another way to check what you have found
- something interesting you have found [pause] or anything else [pause] you decide

Examine these suggestions in terms of the dimensions for optimism. Implicit in these foci is a faith in student ability to overcome obstacles, and work things out for themselves (See Table 6).

Table 6. Implicit messages when groups address the suggested report foci

Suggested report foci	Message
Something you are not sure about and you are hoping another group can help you with	What we don't know now we will be able to work out, even if this time it needs the assistance of others to get there (temporary nature of failure; personal contribution to success; by identifying the specific nature of the present 'failure')
Something you didn't know before and you do know now	Look! We didn't know it before but we worked it out for ourselves (temporary nature of failure; personal contribution to success)
Some pattern you have found	*We* worked it out for ourselves (personal nature of success)
A mathematical argument for why this pattern exists	Mathematical reasons do not have to be provided by the teacher. With perseverance, we can do it! (Perseverance can lead to permanent success 'we know why')
Something interesting you have found. Or anything else. You decide	This is our work and our focus, and if we think something else is important, the opportunity is there to talk about it (personal nature of success; confirming students can follow their own interests)

Valuing the Reports

This is a critical aspect of the reporting process, and one that involves on-the-spot creative thinking. Teacher comments should emphasise the genuine contribution this group's report has made to the mathematical learning for the class. Examples include:

> This group has emphasised for us something that mathematicians need to do when they start a new problem—work out what the problem is all about! This group has been able to identify some of words they see as important, and what they do and don't know about these words at present. That is how mathematicians work
>
> This group says they have found a pattern. That gives us something to think about. You will need to decide (in your groups) whether you agree with this pattern or not. Remember, if you do find a pattern, that is the *beginning* not the *end* because you then want to know '*Why* does this work?

Notice, the teacher did not provide mathematical information, nor restate the student's ideas. The words the first group identified were not restated, and nor was the pattern of the second group. There is no evidence of teacher agreement or disagreement with any of the student 's ideas. The students are in charge of their own decisions about the ideas presented. Sometimes the teacher's comment draws out key mathematical ideas (as questions). For example, groups could have shown the angle above the **x** axis, below the **x** axis, to the right or the left of the line. A teacher comment like:

> Different groups have the angle in different places. Does it matter? Can they all be right? What does the question say? Your groups might want to think about this. Remember you need to explain the reasoning for the decisions you make

The reporting process continually reinforces 'student struggle with adversity', and the temporary nature of this adversity. It also reinforces the permanent nature of success. Students learn that even if they don't know now, they are going to be able to work it out. Students are providing continual proof, to themselves and each other, of their capability to succeed after an initial struggle. It is logical to expect (as the theory implies) that as students work within this type of classroom environment, they become more inclined to explore unfamiliar mathematical ideas in the future.

Conclusions

The student characteristics of resilience, and inclination to pursue novel mathematical ideas, are mutually sustaining—a powerful symbiotic relationship. Glover, Burns, Butler, and Patton (1998) are researching the effects of whole school programs designed to increase student resilience (as a way to prevent or delay the onset of adolescent depression). By increasing the spontaneous and creative nature of student activity in mathematics classrooms, we could contribute to this initiative, as we move closer to developing the characteristics of mathematicians (rather than just disseminating information). These findings are relevant for learners in mixed ability classrooms; the resilient students who explored new mathematical ideas were not all high achieving students. Teachers could use the ideas in this paper to focus their own search for idiosyncratic ways to: (a) increase students' chances to recognise ideas in exploration of rich tasks; (b) facilitate student sharing of their thoughts during their struggles towards a clearer understanding of the mathematics; and (c) emphasise the successes students achieve through personal endeavour. This could help to make schools a more challenging, rewarding, and productive place for young people.

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References

Csikszentmihalyi, M. (1992). The flow experience and its significance for human psychology. In M. Csikszentmihalyi & I. S. Csikszentmihalyi (Eds.), Optimal experience: Psychological studies of flow in consciousness (pp. 15-35). Cambridge: Press Syndicate of the University of Cambridge.

Glover, S., Burns, J., Butler, H., & Patton, G. (1998). Social environments and the emotional well-being of young people. *Family Matters*, 49, 11-16.

Nicholls, J. G. (1983). Conceptions of ability and achievement motivation: a theory and its implications for education. In S. G. Paris & G. M. Olson & H. Stevenson (Eds.), Learning and Motivation in the Classroom (pp. 211-238). New Jersey: Lawrence Erlbaum Associates Inc.

Seligman, M. (1995). The Optimistic Child. Adelaide: Griffin Press.

Williams, G. (1997). Imagine: students approaching mathematics classes with anticipation of excitement, challenge, pleasure and reward. In D. Clarke, P. Clarkson, D. Gronn, M. Horne & L. Lowe (Eds.), *Mathematics: imagine the possibilities* (pp. 43-51). Brunswick, Victoria: Mathematics Association of Victoria.

Williams, G. (1996). Unusual Connections: Maths through investigation. Brighton, Vic: Gaye Williams Publications.

Williams, G. (2002). Autonomous access, spontaneous pursuit, and creative execution: insightful and creative mathematical problem solving. In C. Vale, J. Roumeliotis & J. Horwood (Eds.), Valuing Mathematics in Society (pp. 331-347). Melbourne, Victoria: Mathematical Association of Victoria.

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