SPACES OF LEARNING THAT PROMOTE INSIGHTFUL AND CREATIVE MATHEMATICAL BEHAVIOUR: A THEORETICAL FRAMEWORK[1]

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Gaye Williams

University of Melbourne

g.williams@pgrad.unimelb.edu.au

A theoretical framework (Williams, 2002a, 2002b) formulated to study characteristics of spaces of learning that promote or inhibit insightful and creative mathematical behaviour is described. This framework builds upon the ideas of Krutetskii (1976), Dreyfus, Hershkowitz, and Schwarz (2001), Csikszentmihalyi and Csikszentmihalyi (1992) and Williams (2000b). A tool to display the simultaneous cognitive, social, (Dreyfus, Hershkowitz, and Schwarz, 2001) and affective (Williams, 2002b) elements of mathematically insightful and creative processes is illustrated.

INTRODUCTION

Student alienation with school is an issue of current concern (Marks, 2000) and one solution is to create enhanced spaces of learning where positive affect is linked with new learning (Csikszentmihalyi & Csikszentmihalyi, 1992). The presence of positive affect associated with mathematics learning has been identified in Nicholls' (1983) task-centred rather than ego-centred learning. The characteristics of students undertaking task-centred learning can be seen in instances of learning through 'discovered complexity' as identified by Williams (2000b). This paper discusses a theoretical framework that interconnects the cognitive, social, and affective elements of a space of learning. An analytical tool (Williams, 2002b) that simultaneously displays student cognition, social interaction patterns (Dreyfus, Hershkowitz, et al., 2001) and affective indicators (Csikszentmihalyi & Csikszentmihalyi, 1992; Williams, 2000a) is described.

THEORETICAL FRAMEWORK

Dreyfus, Hershkowitz and Schwarz (2001) are interested in 'abstraction'—an activity of vertical reorganisation of 'previously constructed mathematical knowledge into a new structure' (p. 377). (Vertical refers to a new mathematical structure as opposed to strengthened connection between a mathematical structure and a context ('horizontal')). They have diagrammatically represented a simultaneous display of cognitive activities and social interactions present during the

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abstraction process. Six categories of dialogue were used in the analysis of social interaction—control, elaboration, explanation, query, agreement and attention. Through analysis of the dialogue of participants as they undertake the social process of critical inquiry, the cognitive elements of the process of abstraction were made more visible.

Dreyfus, Hershkowitz, and Schwarz have used three observable nested cognitive elements to study the process of abstraction (*Recognising*, *Building-with* and *Constructing*). Recognising and Building-with are nested within Constructing. These nested elements of abstraction can be described in terms of the categories of a hierarchy developed using Krutetskii's (1976) 'mental activities' [cognitive activities] (Williams, 2000a). The synthesis of these ideas is represented diagrammatically in Figure 1 below (Williams, 2002a). Recognising includes the process of identifying a context in which a previously abstracted entity applies, or recognising a procedure that applies to a new context (Comprehending). Building-with includes using previously abstracted entities as part of several different processes such as: applying a previously abstracted entity in a known context (Applying), or in a new context (Analysing), or applying several previously abstracted entities in a familiar order (Applying) or a different order (Analysing), or finding the interrelationships between two solution pathways that achieve the same goal (synthetic-analysis), or looking at a result from a different perspective to assess the reasonableness of that result (evaluative-analysis). Constructing—mathematically insightful behaviour—is a process of integrating previously abstracted entities to develop a new mathematical insight (Synthesising), progressively checking the consistency of the findings, considering the limitations of the approaches applied, and recognising other contexts in which these new ideas apply (Evaluating).

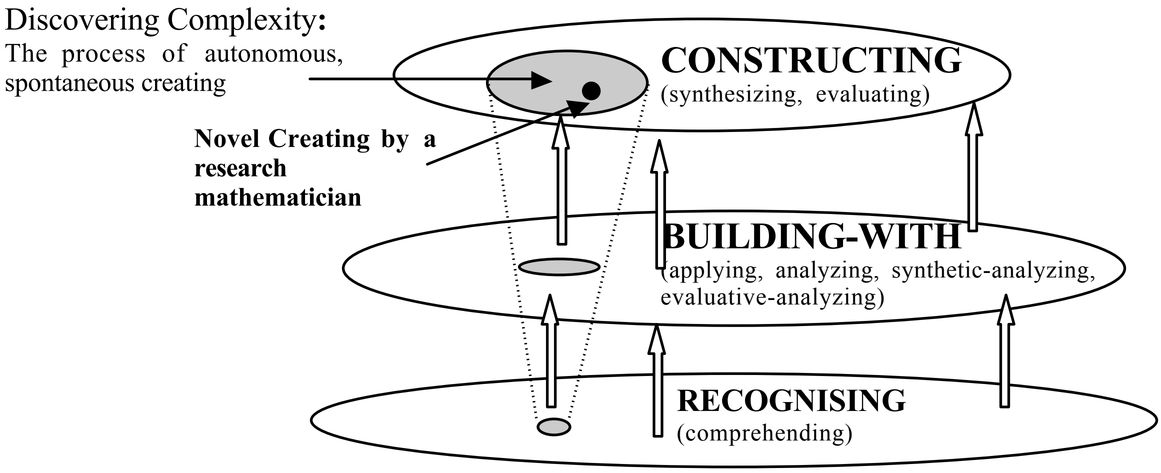


Figure 1. Williams' (2002a,b) representation of the interrelationship between Dreyfus, Hershkowitz, and Schwarz's nested observable cognitive elements of the process of abstraction, Williams' process of discovering complexity, and the process of employing Krutetskii's cognitive activities.

Constructing may occur in a more expert-directed learning culture where suggestions, hints, and corrections are made or constructing may be a more autonomous activity where hints are not provided but questions are asked for the purpose of eliciting more complex thinking (Williams, 2000b).

Figure 1 illustrates the relationship between discovering complexity (see below), the activities of a research mathematician (Chick, 1998), and the observable epistemic elements in the abstraction process. In examining the process of creation of mathematical concepts (a subset of abstraction), Chick (1998) reported the presence of positive affect as strategies, ideas and concepts were synthesised to produce a novel mathematical insight about the complexities discovered. Her description of these processes is consistent with the discovery of complexity: “not only choosing the cues and concepts—and often unexpected cues and concepts—but even the very question” (Chick, 1998, pp.17).

Discovered complexity (Williams, 2000b) occurs during task completion when an individual or group of problem solvers perceive intellectual and conceptual complexities not evident at the commencement of the task. When a complexity is discovered, the individual or group spontaneously formulate a question (in relation to this complexity) that leads to higher level thinking (analysis, analytical-synthesis, evaluative-synthesis, synthesis, and evaluation) in the domain of mathematics. The resolution of the question can lead to abstraction. This process has been found to be associated with high positive affect. Both Nicholls (1983) and Williams (2000b) linked student learning accompanied by positive affect to ‘flow’ (Csikszentmihalyi & Csikszentmihalyi, 1992). Flow is an optimal learning condition that may occur when a person works just above their present skill level on a challenge almost out of reach. Individuals or groups in flow become so engrossed with the task at hand that they lose awareness of self, time and the world.

Key: ➡ The process of discovering complexity (complex cognitive processes employed in conjunction with mathematical ideas and concepts to create novel concepts)

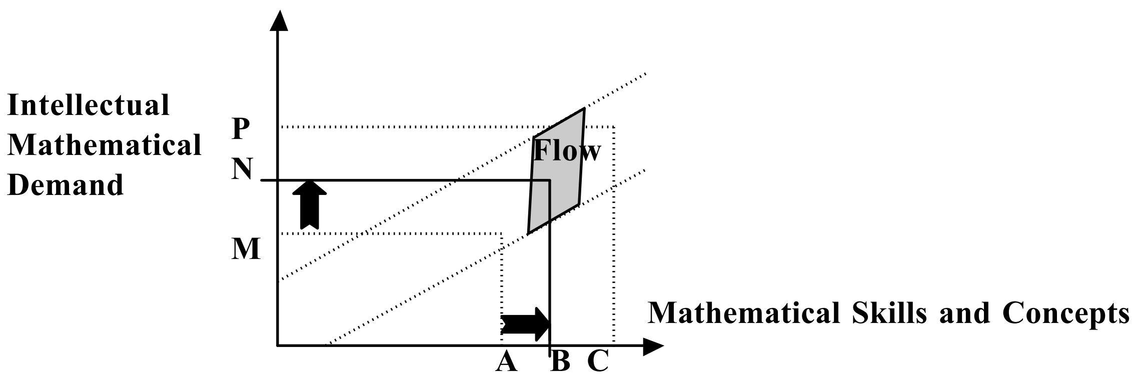


Figure 2. Williams' (2002b) representation of associations between discovered complexity and flow

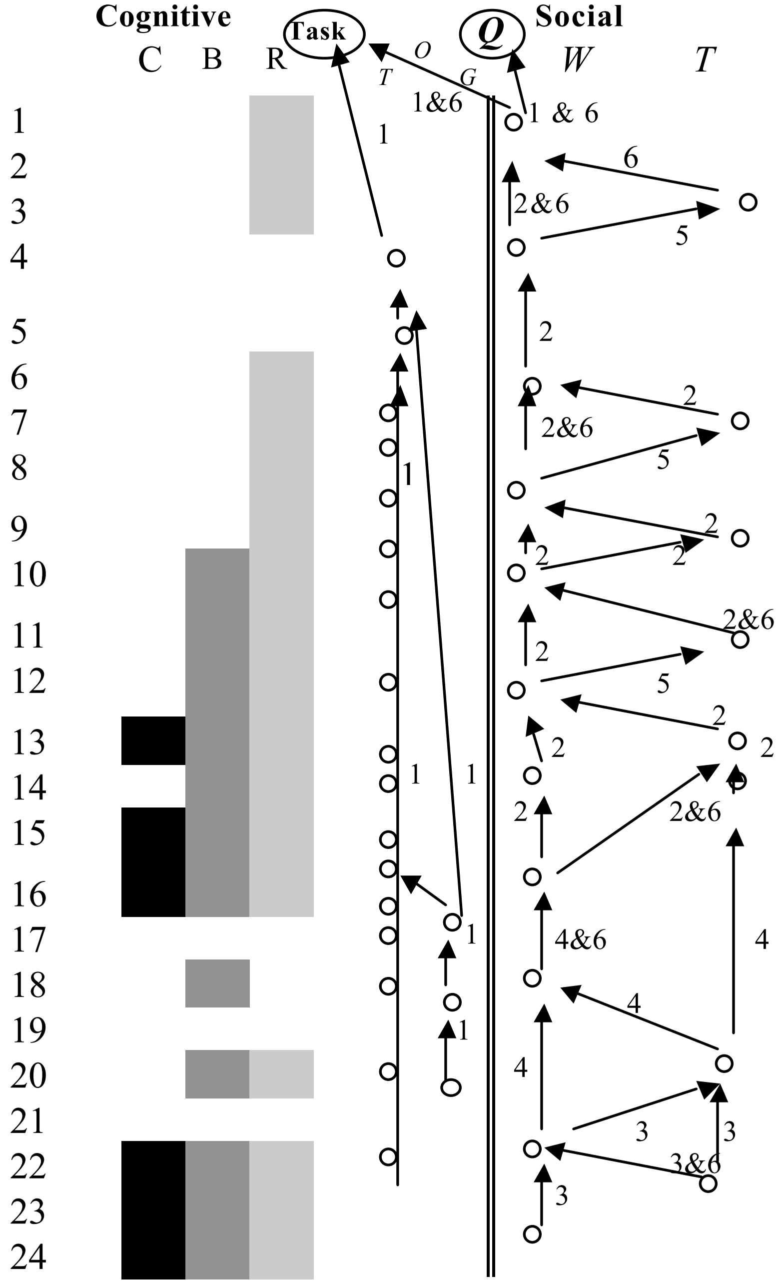
Williams (2000b) illustrated the fit between discovered complexity and the conditions for flow by modifying the schematic representation (Figure 2) developed by Csikszentmihalyi & Csikszentmihalyi (1992). A student’s perceived level of skills and concepts and perceived intellectual challenge comfortably overcome are represented by **A** and **M** respectively. Students in flow are seen as working to achieve a goal represented by a point within the shaded region of flow **(B, N)**. They are working just above their perceived skills and concepts level (horizontal black arrow) on a challenge perceived to be almost out of reach (vertical back arrow). Once this goal is achieved, each student's perceived skills and concepts level has increased to **B** and they can comfortably achieve a challenge of **N**. The shaded region representing flow is now located to the right of **B** above **N** between the parallel lines. To sustain flow would require a discovered complexity that led to a goal represented by a point like (**C, P**). By definition (Williams, 2000b), the mathematical ideas in a discovered complexity are new to all students in the group and the teacher does not contribute new mathematical ideas during the interaction. Where the student group develops new concepts through discovered complexity, the type of student response described is seen to be partially attributable to the task and partly to the pedagogical approach (Williams, 2000a)—both of which form part of the space of learning. Information about a tool developed to assess task potential can be found in Williams (2002a). The process students enact when working with discovered complexity is an insightful process of autonomous, spontaneous, and creative abstraction similar to the activity described by a research mathematician (Chick, 1998).

Analysis of what triggered each part of the dialectic interaction (social elements of query, attention or/and control) and the nature and source of the conceptual artefacts upon which students draw provides evidence of the degree of student autonomy and spontaneity. Attention to whether the dialectic interactions occur in response to earlier dialogue, within the interaction under study, facilitates the identification and analysis of student spontaneity and student autonomy. Interactions are seen to be more autonomous and spontaneous where the students within the pair responded to interactions from within rather than outside the pair. Where autonomous and spontaneous behaviours were exhibited, further analysis could be undertaken to find evidence of creativity. It is recognised that the cognitive artefacts students assemble during the creative process could include strategies, ideas and concepts that rely upon contributions from individuals or other resources, external to the group, prior to this interaction. In relation to an interaction under study, these previous influences contribute to the cognitive artefacts students assemble (Dreyfus, Hershkowitz, et al., 2001). The extent to which an interaction is seen as creative in this study depends partly upon the spontaneity with which students assemble these artefacts and the students' prior knowledge of their relevance. The simultaneous display of affective indicators 'makes visible' associations between engagement with the task and students' autonomy and spontaneity in the process of 'abstraction' (Dreyfus, Hershkowitz et al., 2001).

SIMULTANEOUSLY DISPLAY: COGNITIVE, SOCIAL, AND AFFECTIVE

The analysis tool (Figure 3) used to explore associations between student affect and the creative process of abstraction is a diagrammatic representation displaying the cognitive, social, and affective features of the interaction. Numbers down the left hand side of the diagram represent the line of transcript (a small excerpt of this transcript has been included as part of this illustration).

Key: Cognitive **C B R** (constructing, building-with, recognising). Social **O W T** External // Internal. Affective: **Ey D U P L Ex** (eyes, body, unaware, participate, latch, exclaim)
Numerical codes for social interactions: 1.control, 2. elaborate, 3. explain, 4. query, 5. agree, 6. attention.



Line	Ey	D	U	P	L	Ex
1	√	√		√		
2		√		√		
3	√	√		√		
4	√	√	√	√		
5	√	√	√	√		
6	√	√	√	√		
7	√	√	√	√	√	
8	√	√	√	√	√	
9	√	√	√	√	√	
10	√	√	√	√	√	
11	√	√	√	√	√	
12	√	√	√	√	√	
13	√	√	√	√	√	
14	√	√	√	√	√	
15	√	√	√	√	√	
16	√	√	√	√	√	
17	√	√	√			
18	√	√	√	√		
19	√	√	√			
20	√	√	√	√		
21	√	√	√	√		
22	√	√	√	√		√
23	√	√	√	√	√	
24	√	√	√	√	√	

Figure 3. Tool to aid identification of student autonomy, spontaneity, and creativity in the abstraction process (from Dreyfus, Hershkowitz & Schwarz; 2001; Williams, 2000)

The shaded squares in the first column indicate the inferred presence of epistemic cognitive elements in the process of abstraction for the individual speaking in that line of transcript. The centre column contains a vertical line that separates the small circles representing those within the collaborating pair (William and Talei) from those outside the collaborating pair (Gerard and the teacher). The numbered arrows indicate the individuals to whom the speaker appears to attend and the social interaction category to which those prior statements belong. Arrows that cross the separating line in the centre column suggest less spontaneity and autonomy than when the arrows from members of the collaborating pair are directed to dialogue within the pair. The number of ticks in the grids on the right hand side provides an indication of task engagement. Engagement with a task was analysed through video records of body language indicators of the state of flow (Williams, 2000b). These include: (a) eyes on the task (Ey); (b) pens on the task page or bodies leaning in towards the task (D); (c) unaware of the world around (U); (d) participating in the interaction (P); (e) students building on each other's ideas (latching comments) (L); and (f) exclamations of pleasure (Ex). The symbols EyDUPLEx are used in Figure 3 to represent body language indicators.

By considering cognitive activity in conjunction with social interactions, inferences can be drawn about student creative behaviour. Where students are constructing and the social interaction pattern indicates students are responding only to sources internal to the group, the students display autonomous, spontaneous behaviour that may be creative. The pattern shown by the directions of the arrows from William and Talei (Figure 3, centre column) indicate the pair responded only to each during the interaction. William and Talei displayed numerous indicators of positive affect (Figure 3, right column) as this autonomous, spontaneous pair employed mathematically insightful behaviour whilst constructing a new mathematical insight. In Line 4 the teacher began to organise the next part of the classroom activity (student reports of what they had found) but Talei and William were so focused on their student-initiated activity in response to William's reflection 'There must be something more' (Q) that they appeared unaware of the teacher. They worked to recognise the cognitive artefacts upon which their novel concept relied. The parallel dialogue (teacher and student pair) from Line 4 to Line 24 illustrates these two students' lack of awareness of the world around because all energies where directed to the task at hand. The affective indicators in the right hand confirm the development of a state of flow for William and Talei. Their eyes and pens were on the task and they both leant forward. Both participated in the interaction as they latched their comments to the previous comment as they moved progressively closer to creating a novel concept. The moment of realisation (or insight) was accompanied by an exclamation from William and a hand movement signifying the new understanding from Talei.

The two lines of transcript (below) illustrate the latching process. William is describing what happens to the slope of a graph and Talei builds upon his idea by recognising he is describing the curvature of the graph. Curvature had not been mentioned in this mathematics subject. It is a self-selected cognitive artefact recognised by Talei as she listened to William's description [Key: '{' latch comment to previous comment]

12 William: {’till there when it starts to get more}

13 Talei: {So when the curvature is

As the three students generally worked collaboratively responding to each other's comments, Lines 18, 20, and 22 (where Talei and William do not respond to Gerard's questions) provide further confirmation of the presence of flow.

SUMMARY

A more detailed characterisation of the space of learning to identify other factors that contributed to the student interaction would require exploration of variables such as classroom culture, task characteristics, and the teacher’s introduction to the task. This brief illustrative example demonstrates how this visual tool connects to my theoretical framework thus informing my research. The tool facilitates my identification of autonomous, spontaneous, and creative student abstraction as opposed to abstraction in general. Further work is required to extend the tool to examine the nature of dialectic intervention from an ‘outsider’ who stimulates rather than inhibits spontaneity and creativity. The tool, which presents possibilities for examining such incidents, was designed for use with the video and interview data in the Learners’ Perspective Study of Year 8 mathematics classrooms in 9 countries.

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