DYADIC PATTERNS OF PARTICIPATION AND COLLABORATIVE CONCEPT CREATION:

'LOOKING IN' AS A STIMULUS TO COMPLEX MATHEMATICAL THINKING

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Dyadic interaction is a frequent occurrence in Australian classrooms, where students commonly work collaboratively in pairs. The extent to which such dyadic practice might constitute a coherent pattern of participation has been discussed by Clarke (2001) in an earlier study. Central to the development of such dyadic patterns of participation is the progressive refinement of intersubjectivity between the pair of students, with regard to both content-specific and social meanings. The major focus of this paper is a particular pattern of participation identified by Williams in two student dyads in Year 8 mathematics classrooms. This pattern was characterized by: one student generating a visual display, the second student asking for explanation and not receiving it, and, the second student extracting the underlying structure from within the visual display. It has been suggested elsewhere (Holton & Clarke, 2002) that students can reciprocally scaffold each other's learning activity. This paper describes one particular pattern of participation that appeared to scaffold complex thinking in mathematics.

INTRODUCTION

Working in pairs is common in Australian and American classrooms. Clarke (2001) reported the well-established dyadic practice of Lauren and Karen, whose classroom interactions made a fascinating study: incomplete statements, unclear referents, interruptions, pronoun use, all indicated the existence of a rich and well-established pattern of dyadic practice built around shared purpose and shared meanings. Clarke (2001) argued that this example provided a useful illustration of the pre-existence within the Lauren/Karen dyad not just of intersubjectivity regarding the meanings of the mathematical terms they exchanged during their interaction, but also the pre-existence of established ways of working as a dyad. It was argued in this analysis that the co-construction of these patterns of dyadic practice involved the iterative refinement of intersubjectivity, just as is the case for mathematical meanings (see Clarke, 2001; Cobb & Bauersfeld, 1995; Lerman, 1996; Steffe, 1995; Steffe & Thompson, 2000; and Voigt, 1995). However, in this case, the matter of the meanings related to the individuals' social practice (that is, to their patterns of participation), and the product of this process of iterative refinement was a body of dyadic practice, specific to the Lauren/Karen dyad. Lave and Wenger, in associating learning with participation in practice, assert that "Participation is always based on situated negotiation and renegotiation of meaning in the world" (Lave & Wenger, 1991, p. 52).

It is worth considering a sample interaction between Lauren and Karen (Figure 1).

Turn	Transcript
1	S19: It says how many sheets of graph paper would you need to show one million one millimetre squares.
2	L: To show one million, you know you don't divide it by one hundred, because there's more than a hundred one millimetre squares. I mean you're going to find the area of this.
3	K: What?
4	L: You've got to find the area of this, there's more than one hundred one millimetres.
5	K: That's right. I was doing length by—oh screw that.
6	L: One hundred one millimetre squares.Take length—
7	K: Um, there's how many down here?
8	L: And along that side there is—
9	K: Ten, twenty, thirty, forty, fifty. How many are there down there?
10	L: There's a hundred one millimetres there.
11	L: No, there wouldn't be.
12	K: There wouldn't be, that's not right.
13	L: There'd be two hundred and fifty.
14	K: Yeah.
15	L: Yeah, there'd be two hundred and fifty.
16	K: And we just totally screwed it all—
17	L: Length of graph.
18	K: OK, so it would be length times width [inaudible]
19	L: And uh, two hundred and fifty millimetres. Width—
20	K: What's width?
21	L: That's—
22	K: That's ten, twenty, thirty, forty, fifty, etcetera.
23	L: Eighteen, one hundred and eighty.
24	K: Times one hundred and eighty. OK here we go. Two hundred and fifty times one hundred and eighty equals forty-five thousand. OK, that's forty-five thousand. We need a million. What's a million divided by forty-five thousand and times it by that?
25	L: Hang on, hang on, hang on, hang on. Don't go too fast. OK. Therefore there are forty-five thousand million mm squares.
26	S20: Forty-five thousand million?
27	L: Yeah.
28	S20: Forty-five thousand.
29	K: Twenty-two point two.
30	L: On one piece. Of graph paper.

Figure 1. Lauren and Karen – A dyadic pattern of participation

This episode provides an example of intersubjectivity in practice in a classroom setting. Transcripts of classroom conversations between students who have established patterns of collaborative interaction commonly have some of the features displayed in Figure 1: Sentences are ungrammatical or incomplete; pronouns are used without textual clues as to their referents; and single word utterances are frequent. Communication in this form is only sustainable because the participants share understandings of the referents of the pronouns or key words, and of the processes, actions, or relationships suggested (but not stated) by the sentence fragments and each other's gestures. It is important here to distinguish the overt text represented by the literal transcription above from the implicit text being co-constructed by the participants. The existence of an implicit text can be inferred on the basis that the interaction appeared to be both purposeful and successful.

What was evident in the episode recorded in Figure 1, and in other similar episodes involving Lauren and Karen, was the pre-existence within the Lauren/Karen dyad not just of intersubjectivity regarding the meanings of the mathematical terms they exchanged during their collaborative interactions, but also the pre-existence of established ways of working as a dyad. Lauren's contributions were frequently interpretive or explanatory with respect to the task or a particular procedure. Karen, on the other hand, typically advanced the discussion by asking many questions of Lauren (and, implicitly, of herself) and by verbalising calculations. The co-construction of these dyadic patterns of participation involves the iterative refinement of intersubjectivity with respect to both content-related meanings and with respect to the body of dyadic practice, specific to the Lauren/Karen dyad, that constitutes a particular pattern of participation in the broader activities of the classroom.

By way of contrast, Williams (2001) provided an example of a pattern of participation between a group of three senior secondary calculus students working on an unfamiliar challenging problem. As each discovered complexity arose, the same interaction pattern tended to occur (Fig. 2) and each of these interaction cycles was generally completed in less than a minute. Talei, William and Gerard were generally able to commence the resolution of the focus question associated with each discovered complexity together (see 1. Figure 2).

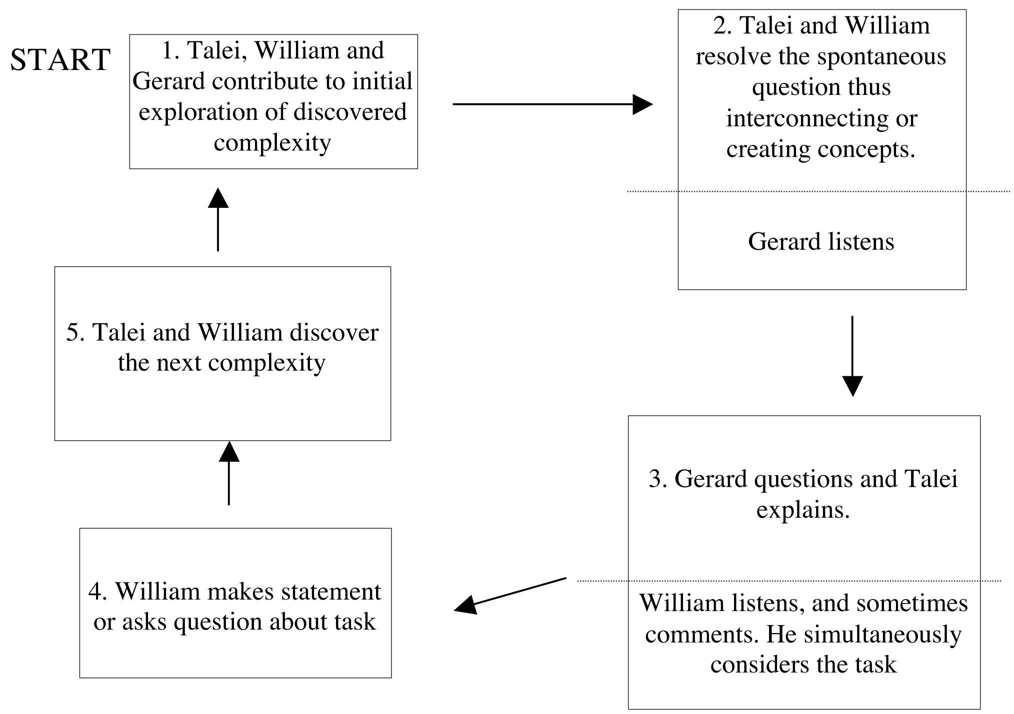

Figure 2. Usual interaction patterns in Williams' collaborative group

Talei and William employed cognitive processes more rapidly than Gerard and were able to draw on a wider variety of cognitive processes that assisted in the synthesis of novel concepts (see 2. Figure 2). Talei and Gerard's interaction (see 3. Figure 2) enabled Gerard to participate in the discovery and initial exploration of another complexity (see 1. Figure 2). The mathematical ideas surrounding the newly discovered complexity were brought to the attention of the group by William (see 4. Figure 2) and the spontaneous question was raised by Talei or William (see 5. Figure 2). The next cycle was then ready to commence.

This triadic pattern of participation hints at several of the ideas central to this paper. As was the case with the Karen/Lauren dyad, progress through the task involved the iterative refinement of intersubjectivity. In the case of Talei, William, and Gerard, their dialogue demonstrated that they created a succession of increasingly more complex interrelated concepts as they successively discovered complexities (Williams, 2000). In addition, the unevenness of the participation in the group discussion served to illustrate the need for negotiation of meanings (and the accompanying refinement of intersubjectivity) among the group members at each stage during the interaction represented diagrammatically in Figure 2. The efficiency with which these negotiations occurred is one indicator of an established pattern of participation. This particular pattern required individuals to 'drop in and out' of the discussion consistent with their 'position' or 'role' within the triad (see Barnes, 2003, for more detail). A priority of Williams' original analysis was to relate the documented pattern of participation to the learning outcomes for the group. As a general principle: Whatever the social grouping that forms the site for our research, some connection must be attempted between the identified pattern of participation and the consequent learning if our research is to inform the promotion of learning in such settings.

The research question relating to the two dyads that provide the primary focus of this paper is: What pattern of participation characterised the interactions of each dyad for the episodes analysed and how did this pattern of participation influence the development of new conceptual ideas?

RESEARCH DESIGN

The data was drawn from Australian data within the broader international Learners' Perspective Study (LPS) into the practices, meanings and learning outcomes of mathematics classrooms as viewed from the perspective of the learner. Each of the nine participating countries has undertaken studies of the classrooms of three teachers recognized by their school community to display 'good teaching practice'. The data from ten to fifteen successive lessons was collected by three video cameras that operated simultaneously in the classroom to display the actions of: (a) the class as a whole; (b) the teacher; and (c) a pair of focus students. Following the lesson, focus students took part in individual audio taped interviews that were stimulated by a mixed image of the video of the teacher (small insert) and video of the focus students. Photocopies of the students' work helped focus the interview and inform the data analysis. Four interviews with the teacher were undertaken throughout the research interval. Greater detail about the Study Design can be found in the methodological discussion by Clarke (2001) or see at the project website: http://www.edfac.unimelb.edu.au/DSME/lps/.

Students selected for discussion in this paper were identified as engaged in concept creation as part of a broader study of factors that promote or inhibit student pursuit of novel mathematical ideas (eg., Williams, 2001). The term 'concept creation' will be elaborated by the examples in this study. An episode demonstrating a pattern of interaction for each student dyad (Leon&Pepe; Eden&Darius) was selected for study. (Using the convention described by Dreyfus, Hershkowitz, and Schwarz (2001), an '&' with no space between the names on either side has been adopted to refer to each dyad). The episodes were selected because of their outward appearance of similarity in the patterns of participation within the dyads, and the apparent differences between these interactions and the interactions described in the literature above.

ANALYSIS AND RESULTS

The patterns of participation between two pairs of Year 8 boys were examined to identify how social interactions promoted student creation of novel concepts. Each dyad was engaged in a similar interaction pattern—one student in each dyad generated a visual display that contributed to a conceptual change for the other member of each dyad. Closer examination of the interaction patterns for each dyad illuminated differences in the patterns of interaction within each dyad particularly in the nature of the triggers to new conceptual understanding; and the nature of the mathematical understanding possessed by the student generating the visual display. The documentation of such patterns of participation has the potential to extend our knowledge about pedagogical approaches that could trigger more complex thinking. Although some evidence is provided about the new concepts developed, this evidence is not provided in detail because the focus of this paper is on the nature of the particular pattern of participation.

The interaction patterns for the two dyads are described and interpreted below.

DYAD 1 (LEON AND PEPE)

This student pair provided more examples of pursuit of novel mathematical ideas than any other dyad in the four Australian classrooms studied. They worked in several different ways during the research period. Leon's description of general collaboration between the pair captured one of their most frequent patterns of participation (more detail about this collaborative interaction can be found in a previous paper (Williams, 2001)):

> Um Pepe and I are sort of [pause] at the same [pause] level of intelligence as well as like level of [pause] you know friendship and stuff [pause] that’s why we work really really well together. ’Cause n- it’s not just one of us doing all of the work [pause] and one of us just getting all of the answers and the other one sort of thinking ‘oh what’s going on here?’ ... it’s almost like we combine [pause] what we are thinking about and we sort of come out with the same answer.

This collaboration between Leon and Pepe is similar to the collaboration between William and Talei (Figure 2) as they completed the resolution of a spontaneous question. New ideas and concepts were generated as a result of the interaction.

The interaction for Dyad 1 that was selected for study in this paper is of a different nature as indicated by Leon's description (Interview):

' … today [the type of collaboration described above] … didn't happen 'cause ... Pepe … was just doing what he wanted to … occasionally we will have things that I want to do that Pepe doesn't really want to do so I will do them and Pepe will just sort of look in occasionally and if I do something wrong [Pepe] will just pick up on it. [This time] Pepe was doing most of the stuff and I was just sort of looking in occasionally going ... oh what's gone on with that? … '

Table 1a. Enriched Transcript: Pepe develops a way to construct the triangle

Line	Lesson Transcript	Leon's interview comments
384s31.	LEON What are you doing?	**Leon:** He's got the compass and I didn't know what he was doing [laugh in voice] with the compass because he's supposed to be ruling straight lines.
384s32.	PEPE Ah, watch yourself. [Pepe has the compass stretched out along the ruler and Leon is leaning in watching. Pepe keeps his eyes on the page and the equipment]	Leon: Sometimes Pepe is a better teacher than the teacher
384s34.	PEPE [to himself] It doesn't make like twenty-eight centimeters [gives back ruler and hesitates then reaches his hand out to take the ruler again. His voice and action indicate he has just thought of an idea] Wait, wait, wait, wait, wait, I still need it.	
384s35.	LEON What the hell are you doing Pepe, are we actually going to do any work or not? [Pepe had drawn one line and is now beginning to use the ruler for a new purpose].	**Leon:** he was doing the circle thing and I didn't know why he was doing the circle thing … that sort of threw me off course.
384s36.	PEPE You reckon we should? [Pepe begins to move the long ruler in an almost arc shape]	
384s37.	LEON You can draw…what are you doing Pepe?	
384s38.	PEPE We don't know how…bloody..ooh, twenty-eight centimetres is. We don't know where the fucking measurement is. [slaps hand down on the page as though indicating both ends of the line] Think about it. [Leon looks at the board for a short period of time. He is not moving, just looking at the triangle on the board as though thinking]	
384s39.	LEON [Leon moves his hands and gestures as he explains his thoughts] Pepe, it's twenty-one centimeters straight up, so go twenty-one centimeters straight up, go in a little bit…[shrugs]	
384s40.	PEPE [Pepe looks at the board and at Leon's hand movements. He then bends over the page again as he speaks] Let me do it my way.	
384s41.	LEON Alright you do it your way then. [Leon turns and leans over to talk to Ellen and Servia again]	
384s42.	LEON [to Servia] Yay…(…) [Leon's off-task talk to the girls continues through to 384s45]	
384s45	PEPE [Pepe holds one end of the ruler as a pivot and swings the other end in an arc on the page. He looks up at the board then scratches his head and turns to Leon] Leon, What does that other one say? On the corner.	

The interesting aspect of the episode chosen for study (Table 1a,b) is that, as noted by Leon, despite their well-established collaborative practice, in this instance Pepe did not explain when asked. Pepe responded 'WATCH [FOR] YOURSELF!'. This comment triggered Leon's intense focus on Pepe's page and subsequent ‘extraction’ of a generalization from Pepe’s diagram. The context is described below and an enriched transcript of the episode under study is provided (Table 1a,b). This transcript includes descriptions of body language, and relevant excerpts of the student interviews (placed beside the lines of lesson dialogue to which they refer). The numbers beside the lesson transcript indicate the line number of the dialogue from the teacher camera. Where students engaged in private dialogue, not captured on the teacher tape, the line number at which this dialogue commenced is followed by an 's' and another number representing the line number within this student exchange. (Eg., 384s31 is the 31st line of talk between the dyad that commenced at Line 384 of the teacher tape).

The context of the lesson: At Line 315 in Lesson 12 the teacher organised students to work in pairs to find the area of a triangle. The scalene triangle allocated to Leon&Pepe contained three pieces of information—the three side lengths. The teacher had assumed students would know how to construct such a triangle (teacher interview) but Pepe and Leon both confirmed this was an unfamiliar problem for each of them (student interviews). Pepe was determined to use his own method (constructing the triangle and counting the squares) to find the area of the triangle. Leon recognised and puzzled over the reasons behind a faster method for finding the area (related to halving the area of the rectangle that enclosed the triangle) whilst he simultaneously engaged in off-task talk with some of the girls (Williams & Clarke, 2002). Pepe's intense interest in the construction process is described below by Leon:

> Leon '… um ... I’m about to do something to it [the paper provided to draw the triangle] and he ... grabs it off me. But he actually starts doing *work* ... on it.'

The episode described (See Table 1a and Table 1b) relates to Pepe's construction of the triangle and Leon's 'looking in' to 'extract' his own generalized understanding of this construction process. Leon justified his 'looking in' behavior (as opposed to joining in):

> Leon '… and um ... but I end up understanding it. If I am sitting there for the whole lesson asking what are you doing this for? What are you doing this for? Neither of us will understand it. If I look in occasionally and like see what he has done like you know twenty or thirty seconds maybe um I understand what he is doing …'

As Pepe developed a novel sequence of procedures to construct the triangle, Leon occasionally asked Pepe what he was doing [Lines 348s31, 35, 37] but Pepe did not explain. In his interview, Leon commented on the part of the video where Pepe responded "Watch [for] yourself" [Line 384s32]:

> Leon Sometimes Pepe is a better teacher than the teacher.

This comment suggests Leon's preference for working things out for himself. This inferred preference was supported by Leon's classroom behavior and his interview. The only time Pepe provided any information for Leon was in Line 384s38 when (partly due to Pepe's own frustrations with implementing his new ideas) he gave a précis (including expletives) of what Leon needed to think about.

Pepe enlisted Leon’s co-operation twice during triangle construction (to hold the end of the ruler as pivot and to read a number from the board) [Table 1a, Lines 384s35, Table 1b, Line 425s10].

Table 1b. Pepe commandeers Leon's cooperation in the construction process

Line	Lesson Transcript	Leon's interview comments
425s1.	LEON [from off screen as he consulted the board to find the measurement of the third side] Pepe twenty-three.	
425s2.	PEPE Twenty-three? [Pepe leaned over his sheet again intent as he worked with the third side of the triangle (23cm). Leon turned to talk to the girls and continued off-task talk until 425s10]	
425s3.	LEON [returned to seat and swung his head backward and forward a couple of times from Pepe to Ellen] [to Ellen] Did you make those yourself like that chick from the ‘Great Outdoors’, or whatever it is?	
425s10.	PEPE Leon. [Leon turned away from the girls and more towards Pepe] Make sure that, make sure that twenty-eight always stays on … [Pepe realizeed Leon had not turned to the page to watch the instructions in detail. He stopped the explanation and stopped moving the arc on the page and turned to gain Leon’s attention] lookit idiot. [hits Leon’s face]	
425s11.	LEON Ow.	
425s12.	PEPE [Leon now leaned towards the page and watched Pepe] Like keep…always keep twenty-eight like there. Alright.	**Leon** and then he’s got the ruler and made me hold it to twenty eight
425s13.	LEON Alright. [held one end of ruler and watched closely to keep the ruler as Pepe requested. Pepe slid the ruler in an arc, marking a curve. Once this was completed, Leon continued to lean over and watch as Pepe put the ruler along the base line and from the base line to the arc]	**Leon** … and he’s gone all the way around it ‘what are you doing that for?’ and he’s gone ‘Just watch’ and I’ve gone ‘oh ... so you can get the angle that is sloping down?’ and Pepe has gone 'yes exactly' that was where I understood it

At Line 425s13 in the lesson Leon reported (in his interview) dialectic dialogue between himself and Pepe that does not appear in the lesson transcript:

> Leon: I’ve gone ‘oh ... so you can get the angle that is sloping down?’ and Pepe has gone 'yes exactly'.

It is unclear whether this dialogue was too soft to hear on camera or whether Leon 'heard' this dialogue internally and remembered it as an actual conversation.

Discussion of Leon&Pepe dyad: Leon began the episode with no understanding of the process of constructing a triangle when three sides are given. He saw Pepe's actions with the compass as just fooling around. When Pepe commanded Leon to watch and think, Leon described his progressive development of understanding of the construction process Pepe utilized. In this case, Leon had developed a new concept 'how to construct triangles when three sides are given' by 'looking in' on the construction Pepe was generating. Leon's interview comments indicated he eventually created the concept that underlay Pepe's construction process:

> It was ... was a little bit gradual and then it was like a little bit at the end ... just popped in my head

Of interest is Leon's comment about Pepe as a teacher. Pepe's way of interacting—where he didn't tell—suited Leon's preferred way of working. Pepe's comment and Leon's response to it suggest the usefulness of a pedagogical approach where students have the chance to think for themselves. More information about Leon's preferred ways of working, cognitive activities, and classroom interactions are described in detail in a previous paper (Williams & Clarke, 2002).

DYAD 2. EDEN AND DARIUS

Eden and Darius frequently sat together and helped each other with the mathematical tasks. The interactions within the Eden&Darius dyad were generally for the purpose of peer tutoring or reciprocal scaffolding (Holton & Thomas, 2001) to consolidate mathematical ideas they had previously been exposed to. This interaction pattern was similar to the interaction between Lauren&Karen described earlier.

The episode for Dyad 2 reported here was of a different nature. It occurred during Lesson 6 when students explored an unfamiliar challenging problem through a computer game. Darius and Eden belonged to different student pairs on this occasion but an interaction was still evident for the dyad of Eden&Darius during this lesson. Eden's computer screen was not in view in Lesson 6 because Eden was a focus student in Lesson 8 not Lesson 6. Darius who sat beside Eden was a focus student in Lesson 6. In Eden’s interview after Lesson 8 he showed he had interconnected ideas about linear functions (gradient, intercepts, and the 'step in Y values') that had not been explicitly interconnected in class.

The context: The computer application on which the students worked in Lesson 6 consisted of a Cartesian plane with 13 'globs' that at first appeared to be randomly placed. The aim was to hit as many globs as possible in one shot by generating linear equations. The students were in the computer room working in pairs; Darius and Eden were in different pairs positioned adjacent to each other almost within easy hearing distance. Their chairs were on wheels and the space between them was so small that they frequently wheeled their chairs across to look at each other's computer screens. Although Eden was not visible on screen, he could be seen when he wheeled his chair over to inspect Darius's screen and heard from his position at his own computer.

This episode focuses on Eden's pursuit of how to position diagonal lines. The parts of this episode captured on video occurred during three different time intervals in Lesson 6 (Table 2a,b). The first mathematical interchange between Eden and Darius occurred twelve and a half minutes into the lesson and lasted less than a minute (Interval 1: Eden asks Darius about 'lines that go on an angle'). The second mathematical interchange occurred approximately 18 minutes into the lesson and lasted approximately 2 minutes (Interval 2: Eden uses substitution to test a point) and the third interchange occurred 26 minutes into the lesson and lasted more than 3 minutes (Interval 3: Eden creates his own concept). The lines of transcript that relate to these three time intervals have been numbered consecutively to facilitate the following discussion.

Table 2a: Eden's focus on 'lines that go on an angle (Interval 1).

(Time in Min.Sec) Line Number.	Lesson Dialogue	Interview comments related to episodes.
Interval 1	Eden asks Darius about lines on an angle	
(12.30) 1.	Eden: I don't get this.	
2.	Darius: You have to shoot them	
3.	Eden: I know but … [Eden looks at the diagonals Darius has on the screen] What's the rule for that? … That's the sort of angle …	Eden: Like last last lesson [Lesson 6] on the computer with the green globs I sort of used them but I did not know what exactly they were … you do so and so x plus 2 …
4.	Darius: Two X plus three- you have to do something X plus something- you can't just do X.	Darius: 'First … we were just trying the way … she's … told us … [now] I am just trying … different combinations …'
5.	Darius: … [Darius now has quite a few lines on his page] See it's easy!	
6.	Darius: [to teacher] But how do you actually get them where you want them? Because I'm just tapping anything in.	Darius: ' … you didn't know at first … so um basically you just have to *see* … just type in anything … and then see where it goes [to find the position of lines]'

In his interview Darius stated he was interested in achieving 'hits', not in working out why that graphs are positioned as they are. Eden looked at these graphs and wanted to know how to position them because they possessed the type of angle he wanted to achieve [Table 2a, Line 3]. Darius stated the general rule the teacher had told them and explained how he used this for his trial and error by continually adjusting the numbers in the equation to get 'hits'. In doing so he did not provide any information about what the numbers did to the position of the graph [Table 2a, Line 4]. Darius' comment to the teacher in Table 2a, Line 6 confirmed his trial and error approach.

Eden paused as he quietly looked at Darius' screen [Table 2b, Line 7]. Darius gave numerical information about a line he knew would hit a particular glob [Table 2b, Line 8]. The information given was two numbers (the coefficient of X and the constant). Conceptual development is evidenced by Eden's exclamation 'oh I get it' [Table 2b, Line 9]. It appeared he had just recognized he could check whether a point lay on a line by using substitution. His comment appeared to be a substitution into the equation y = -2x + 2 to get 4 when X is minus 1. This suggested Darius was pointing to the point (-1,4). This recognition of substitution as a valuable strategy (triggered by Darius' attention to a particular point) appeared to contribute to Eden's later breakthroughs.

Table 2b. Eden's focus on 'lines that go on an angle (Interval 2 and Interval 3).

(Time in Min.Sec). Line Number.	**Lesson Dialogue**	**Interview comments or occurrences in Lesson 8 related to this episode.**
Interval 2	Eden uses substitution to test a point	
(17:40). 7.	Eden: [comes over to Darius's screen]	
8.	Darius: [points to the screen] Negative two-positive two will get you that dot.	
9.	Eden: Oh I get it- if you do two plus two is four	
(19:40) 10.	Darius: Yeah- now I know how to actually get the stuff.	
Interval 3	Eden creates his own concept	
Approx 26 mins 11.	Darius: Oooh! I've got it- [looks at everyone around] I can go X …	Darius: ' … she told us … [how to write line graph equations] … and then I was just moving it a bit according to that …'
(26:41) 12	Eden: How the hell did you get nineteen?	
13	Darius: [looks at several lines lined up along a diagonal] I wonder how you can get that?	
(27:53) 14	Martin (Darius' partner): Oh yeah! [as he shoots a glob on a diagonal]	
15	Darius: Hu! [as he misses a glob]	
16	Eden I don't know how you get that. [Darius types on]	
17	Eden [watches, looks back at his screen, and goes back to his own screen]	Towards the end of Lesson 8 the teacher introduced 'steps in Y' by asking about a specific example. Eden volunteered the correct answer immediately suggesting he was familiar with these ideas.

In Table 2b, Line 11, Darius exclaimed loudly when he hit more than one glob with a line. He then looked and waited for others to see he had achieved a new highest score. Once Eden saw

Darius' new score, he asked how he achieved it [Line 12]. Darius did not answer; instead Darius wondered out loud how to make a line go through particular points on his screen. Eden stood and looked at Darius' screen for more than fifteen seconds without making a sound. He then returned to his own screen and did not speak to Darius again about this work.

It appears this reflection about Darius' screen in Line 17 may have triggered Eden's development of an understanding of a property of linear graphs (the increase in Y value remains the same for each increase of one in X). This inference is drawn from several occurrences: (a) Eden said he worked out these ideas in Lesson 6 (Interview); (b) Eden's question to Darius in Table 2, Line 16 suggests he has not recognized the idea at this stage; (c) Line 17 is the last time Eden interacts with Darius or his computer screen in relation to these diagonal lines; (d) Eden answered the teacher's specific question in Lesson 8 (about a line and it's step in Y value) before the teacher provided an example.

During Eden's interview (at the end of Lesson 8) he demonstrated he had worked out how to use the substitution process to work backward from knowing the co-ordinates to knowing the equation:

> Eden: **X** would be –2, -1, 0, 1, and 2, **Y** starts off on minus 3 with the hand out you had to work out what Y was which is –2, -1, 0 and 1, the rule is ah um **Y** equals **X** minus 1.

Eden developed a conceptual understanding of the properties of linear equations that interconnected ideas in ways not discussed in class. It appears Eden's reflections about the lines on Darius' computer screens in Line 17 (Table 2b) triggered the process that led to the interconnections between substitution to test a point, constant increase in Y value, and the use of constant increase in Y in combination with substitution to find the equation to a line when points on the line are known.

This dyad provides an interesting example of another case of 'looking in'. Eden looked in on the screens generated by Darius' trial and error and reflected upon the reasons behind the positions of the graphs. This example provides further evidence of conceptual development achieved through 'looking in' as another student undertakes an activity. The difference in this case is that Darius, who provided the stimulus for Eden's conceptual development, was only developing empirical not theoretical generalizations (Davydov, 1990); Darius was only interested in how to achieve the visual image he wanted. He did not consider the underlying mathematical structure. Eden used Darius' generation of visual images through trial and error as stimulus for concept creation.

DISCUSSION AND CONCLUSIONS

There are similarities between the patterns of participation exhibited by each of the dyads described. Dyad 1 (Leon&Pepe) and Dyad 2 (Eden&Darius) both demonstrated the pattern 'looking in' which is characterized by one member of a dyad extracting a new concept as a result of reflecting upon a visual display generated by the other dyad member. The similar characteristics exhibited by each dyad included:

- One member of each dyad (Pepe, Dyad 1, and Darius, Dyad 2), ‘the generator’, generated a new visual stimulus.

- The second member of the dyad (Leon, Dyad 1, and Eden, Dyad 2), 'the extractor' reflected on the underlying mathematical ideas in the dynamic visual representation produced by the generator. The word extraction has been selected to describe the process of development of these new ideas because the extractor selected the key attributes (essence) from the visual representation rather than the generator drawing attention to these attributes.
- The extractor asked questions of the generator 'What are you doing Pepe?' (Leon) or 'I don't know how you get that?' (Eden)
- In most instances, the generator did not respond to these questions. For each dyad, there was one occasion when the generator responded orally to the extractor's question. Pepe (to Leon) 'ah watch [for] yourself' and Darius to Eden '[points to the screen] Negative two - positive two will get you that dot'.
- This oral response did not answer the question but appeared to be one of the triggers to the 'extraction' process.

The introduction to this paper argued the case for the existence of coherent, regular and structured dyadic patterns of participation practiced by pairs of students in the classroom and exemplified by the Lauren&Karen dyad and, by way of contrast, in the cyclical interactions between Talei, William and Gerard. The discussion of the two dyads Leon&Pepe and Darius&Eden provided a much more detailed description of one particular type of interaction, constituent of the dyadic pattern of participation established for each dyad.

The first step towards improving a process is to understand it. As we learn more about how to elicit complex thinking in the mathematics classroom, it becomes more important to find ways to encourage students to share their ideas for the purpose of scaffolding the learning of others. Given the prevalence of 'working in pairs' in Australian and American classrooms, and the potential this form of classroom participation has for the devolution of responsibility for the generation of knowledge (see Clarke, 2003), the optimisation of this practice should be given priority in classroom research and classroom practice. Pedagogical approaches that provide opportunity for autonomous and spontaneous student work on unfamiliar problems in an environment where dynamic visual student displays are used as part of the solution process may increase the potential of eliciting complex thinking from students. Essential to the promotion of the conditions of autonomy and spontaneity that we believe promote students' engagement in complex mathematical thinking is the recognition of the function of dyadic patterns of participation such as those discussed in this paper. Given the prevalence of pair work in our classrooms, the better such patterns of participation are understood, the better equipped we are to provide optimal classroom conditions conducive to productive dyadic interaction.

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